CHEG 355 Transport Phenomena I
Final Exam

December 18, 2003

Closed Books and Notes
Second problem counts double!

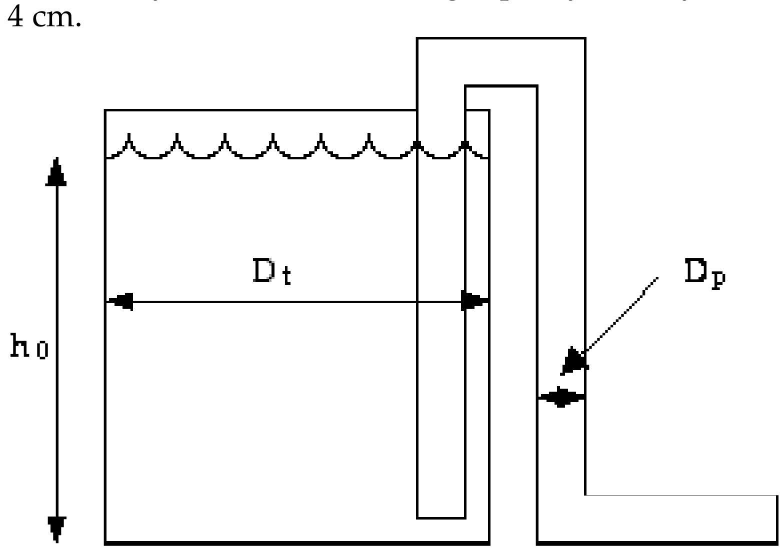
Problem 1. (20 points) You are to design a siphon which will be used to drain a tank, as depicted below. We want to figure out what diameter pipe needs to be used for the siphon to drain the tank to the bottom in the desired time. Because the driving force (the height of fluid in the tank) varies with time, so does the velocity, the Reynolds number, and hence the friction factor. For purposes of estimation we will ignore the friction factor variation, and just use an approximate average value to be determined iteratively. Thus:

a). Set up the equation governing the change in the height of the fluid in the tank as a function of time.

b). Taking the friction factor to be constant, solve the differential equation for h(t) and determine the time for the height to go from an initial value h_0 to zero. Use variables!

(hint: $\frac{1}{h^{1/2}} \frac{dh}{dt} = 2 \frac{d\left(h^{1/2}\right)}{dt}$)

c). For a tank 2 meters high and 2 meters in diameter, what diameter pipe should you use so that it drains in 1 hour? Take the length of the siphon to be 5 meters. You will have to solve this iteratively, however it converges pretty fast if you take an initial guess of around 4 cm.



You may find the following expressions useful:

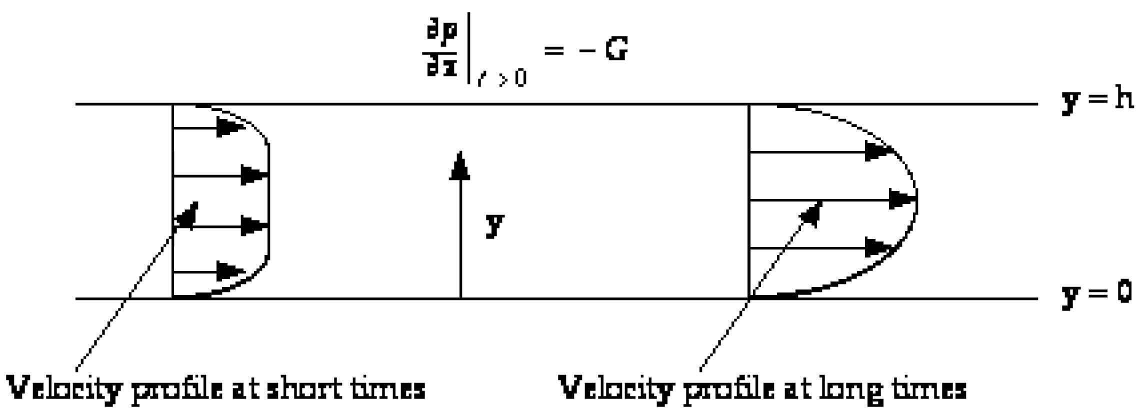
$$h_L = \frac{\langle u \rangle^2}{2g} \Sigma K + 4 f_f \frac{L}{D} \frac{\langle u \rangle^2}{2g}$$

$$f_f = \frac{16}{Re} \;;\; \text{Re} < 2100 \qquad\qquad f_f \approx \frac{0.0791}{Re^{1/4}} \;;\; 3000 < \text{Re} < 10^5$$

$$\frac{1}{\sqrt{f_f}} = 4.0 \, log_{10} \left(Re \quad f_f \right) - 0.40 \;;\; \text{Re} > 3000$$

Fitting	K value
sudden contraction	0.45
sudden expansion	1.0
90° elbow	0.90

Problem 2). (40 points) Unsteady unidirectional flow. Consider the **uni-directional time-dependent start-up flow** depicted below. Initially the fluid is at rest in a channel of width h. At time t = 0 we impose a pressure gradient dp/dx = -G (a constant) in the x-direction. This causes the fluid to accelerate, eventually reaching some asymptotic velocity distribution. In this problem we use scaling to examine all the interesting parts of the problem.



a). The starting point: Write down the governing differential equation valid for all times and associated boundary conditions, crossing out all the terms which are zero. Do not include gravity.

b). Long times: Render the equations dimensionless, using some undetermined timescale t_c for time. Show that for very large t_c (e.g., if we wait a long time), inertia no longer matters, and obtain the appropriate scaling for the asymptotic velocity. About how long do we have to wait?

c). Long times: Solve for the asymptotic velocity distribution at long times, and determine the shear stress at the lower wall (hint: the velocity distribution should be -really- familiar by now!)

d). Short times: Rescale the velocity for very short times, and show that now the viscous term becomes unimportant. How short does t_c have to be? What is the velocity scaling for short times?

e). Short times: Solve for the now time-dependent velocity profile. (Hint: It's really simple, and you should remember what happens to the B.C.'s when you throw out the viscous term...)

f). Short times: Near each wall we will develop a boundary layer at short times. While there's one at both walls, we'll just look at the one near $y = 0$ for simplicity. Rescale the y coordinate and determine the boundary layer thickness as a function of t_c.

g). Short times: Using either the results of part f, or via simple affine stretching, show that the short time boundary layer problem admits a similarity solution. Give the similarity rule and the similarity variable in canonical form, and determine the time dependent wall shear stress to within some unknown multiplicative constant (e.g., the solution of the transformed differential equation that you don't have time to get).

h). All times: Using the results of (b) and (g), sketch up the wall shear stress as a function of time (rendered appropriately dimensionless) in the two asymptotic limits. Note that there is a gap between the two asymptotic solutions which must be bridged via another solution technique. It's actually fairly straightforward to get that one too, but we won't worry about that here... Do point out the domain of validity of the two solutions, however.

Problem 3. (30 points) Pump Curves / Additional Readings / Short Answer:

The first five questions refer to the pump curve on the last page:

1. It is desired to pump 45 liters/sec from a pond to an elevation of 20 meters. If we neglect all frictional losses (say we use a really fat pipe!) is the pump CP100 recommended for the job?

2. What is the RPM required to do the job?

3. What is the work done <u>by</u> the pump <u>on</u> the fluid?

4. What is the efficiency of the pump at the operating conditions?

5. How far up the hill from the level of the pond can we put the pump? (Again, neglect frictional losses) (Note: 1atm ≈ 10.3 m water)

6. At high Re, drag principally results from:

A. Potential Flow
B. Boundary Layer Separation
C. Skin Friction
D. Turbulence

7. Why do dimpled golf balls and fuzzy tennis balls have less drag than their smooth counterparts? One sentence, please.

8. Why did the Tacoma Narrows Bridge collapse? One sentence, please.

9. Ensuring that the NPSHR is met for a pump installation will help prevent:
A. Cavitation in the pump
B. Damage to pipes and fittings due to vibrations
C. Reduced pump performance
D. All of the above

10. Write down the Navier-Stokes equations in index notation.

11. The displacement thickness is defined as:

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy$$

Provide a brief physical interpretation of this quantity.

12. The momentum thickness is defined as:

$$\theta = \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

Provide a brief physical interpretation of this quantity.

13. Experimentally, how can you most accurately calculate the shear stress on a plate in boundary layer flow at high Reynolds numbers from the velocity profile?

14. Give a physical description of the Reynolds stress (e.g., where does it come from?).

15. Just as in the case of turbulent momentum transfer where we define a turbulent kinematic viscosity ν_t, so in the case of turbulent energy transfer we can define a turbulent thermal diffusivity α_t. The turbulent Prandtl number is the ratio of these quantities ($Pr_t = \nu_t \,/\, \alpha_t$). What is its approximate magnitude and why?