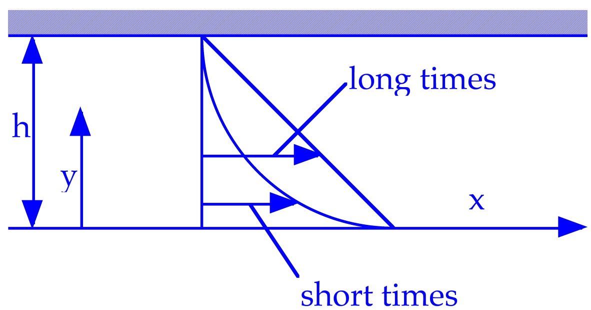
CBE 30355 Transport Phenomena I
Final Exam

December 13, 2017

Closed Books and Notes

Problem 1. (20 points) Unsteady unidirectional flow. Consider the **unidirectional time-dependent start-up flow** depicted below. Initially the fluid is at rest between two plates separated by a distance h. At time t = 0 the lower wall has a stress F/A applied in the x-direction, causing it to move. The upper wall is fixed (no motion). In this problem we use scaling to examine all the interesting parts of the problem.



a). The starting point: Write down the governing differential equation and boundary conditions valid for all times and associated boundary conditions, crossing out all the terms which are zero. Do not include gravity. Don't forget how F/A yields the boundary condition at the lower wall!

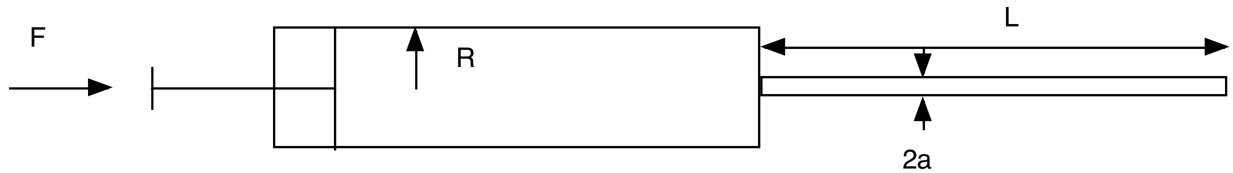
b). Long times: Render the equations dimensionless, using some undetermined timescale t_c for time. Show that for very large t_c (e.g., if we wait a long time), inertia no longer matters, and solve for the asymptotic velocity distribution (which should be *very* familiar). About how long do we have to wait for this solution to become valid?

c). Short times: For very short times the length scale should no longer be the gap width, but instead should be some boundary layer thickness δ. By rescaling the equations in this limit, how do δ and the velocity scale with t_c?

d). Short times: Using either the results of part c, or via simple affine stretching, show that the short time boundary layer problem admits a similarity solution. Give the similarity rule and the similarity variable in canonical form, and determine the time dependent velocity of the lower wall to within some unknown multiplicative constant (e.g., the solution of the transformed differential equation that you don't have time to get).

e). All times: Using the results of (a) and (d), on the same plot sketch up the velocity of the lower wall as a function of time (rendered appropriately dimensionless) in the two asymptotic limits. Show the domain of validity of the two solutions.

Problem 2. (10 points) High Re Flow. Consider the syringe depicted below. Even though needle diameters are small, velocities (and hence Re) are often very high. It is desired to empty the syringe of volume V by applying a force F to the plunger of radius R. The fluid empties through a needle of radius a and length L.



a. Ignoring all frictional losses, calculate the velocity and Reynolds number in the needle and the emptying time. Take the following values: $F = 10^5$ dynes, R = 0.5 cm, V = 10 ml, a = 0.08 cm, L = 4 cm, and use the properties of water.

b. Now improve your result for the velocity, Re and emptying time by including frictional losses in the needle, and a K value of 0.45 for the reentrant entrance to the needle (e.g., sudden rather than smooth contraction). The formulas below will be useful. Remember that iteration will be required, but it should converge really fast!

c. How would you change your approach to this problem if you were trying to squirt out glycerin ($\rho = 1.26$ g/cm^3, $\mu = 14$ poise) instead of water? Be brief, but quantitatively justify your approach!

$$h_L = \frac{\langle u \rangle^2}{2g} \sum K + 4 \; f_f \frac{L}{D} \frac{\langle u \rangle^2}{2g}$$

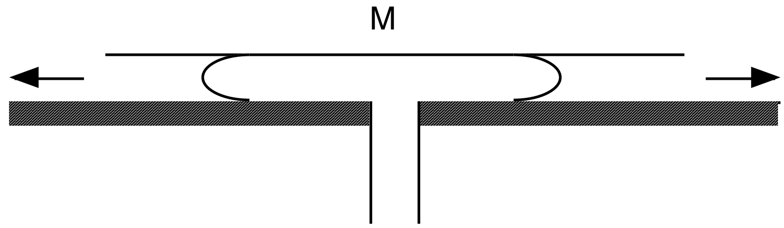
$$f_f = \frac{16}{Re} \; ; \; \text{Re} < 2100 \qquad\qquad f_f \approx \frac{0.0791}{Re^{1/4}} \; ; \; 3000 < \text{Re} < 10^5$$

$$\frac{1}{\sqrt{f_f}} = 4.0 \log_{10}\left(\text{Re}\sqrt{f_f}\right) - 0.40 \; ; \; \text{Re} > 3000$$

Problem 3. (10 points) High Re Flow: Crushing Cans. In class last Thursday Pat and Jack demonstrated the use of vapor condensation to crush a soda can. In this demonstration the can is filled with water vapor, turned upside down, and quenched in a pool of cold water. In this problem we determine how fast we have to condense the vapor to make it work.

If the hole in the can (now immersed in the water) of area A is sufficiently small, the pressure in the can will become very low and the can will collapse. The idea is that if a can of volume V filled with vapor is quenched in some time T, then the pressure inside will decrease if there is enough resistance to the water filling the can through the hole and replacing the condensing vapor. If the crushing pressure is ΔP, develop an expression for the required condensation time.

Problem 4. (20 points) Lubrication. A plate of radius R is supported by the radial flow of a viscous liquid supplied through a hole in the center at a volumetric flow rate Q as depicted below. If the mass of the plate is M, determine the gap width h between the two surfaces.



Note: recall from Calc 2: $\int r\ln(r)\,dr = \frac{1}{2}r^2\ln(r) - \frac{1}{4}r^2$. Also, the diameter of the hole supplying the lubricant doesn't matter, as long as it is small (although it does affect the pressure in the supplying pipe to get a particular flow rate, of course).

The following equations *may* be helpful (particularly if you circle the dominant terms in the lubrication limit!):

$$\frac{1}{r}\frac{\partial(r\,u_r)}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} = 0$$

$$\rho\left(\frac{\partial u_z}{\partial t} + u_r\frac{\partial u_z}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta} + u_z\frac{\partial u_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2}\right] + \rho g_z$$

$$\rho\left(\frac{\partial u_\theta}{\partial t} + u_r\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta} + u_z\frac{\partial u_\theta}{\partial z} + \frac{u_r u_\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(r\,u_\theta)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{\partial^2 u_\theta}{\partial z^2} + \frac{2}{r}\frac{\partial u_r}{\partial \theta}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta} + u_z\frac{\partial u_r}{\partial z} - \frac{u_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(r\,u_r)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial u_r}{\partial \theta} + \frac{\partial^2 u_r}{\partial z^2}\right] + \rho g_r$$

Problem 5). (20 points) Pump Curves / Short Answer:

The first five questions refer to the pump curve on the next page:

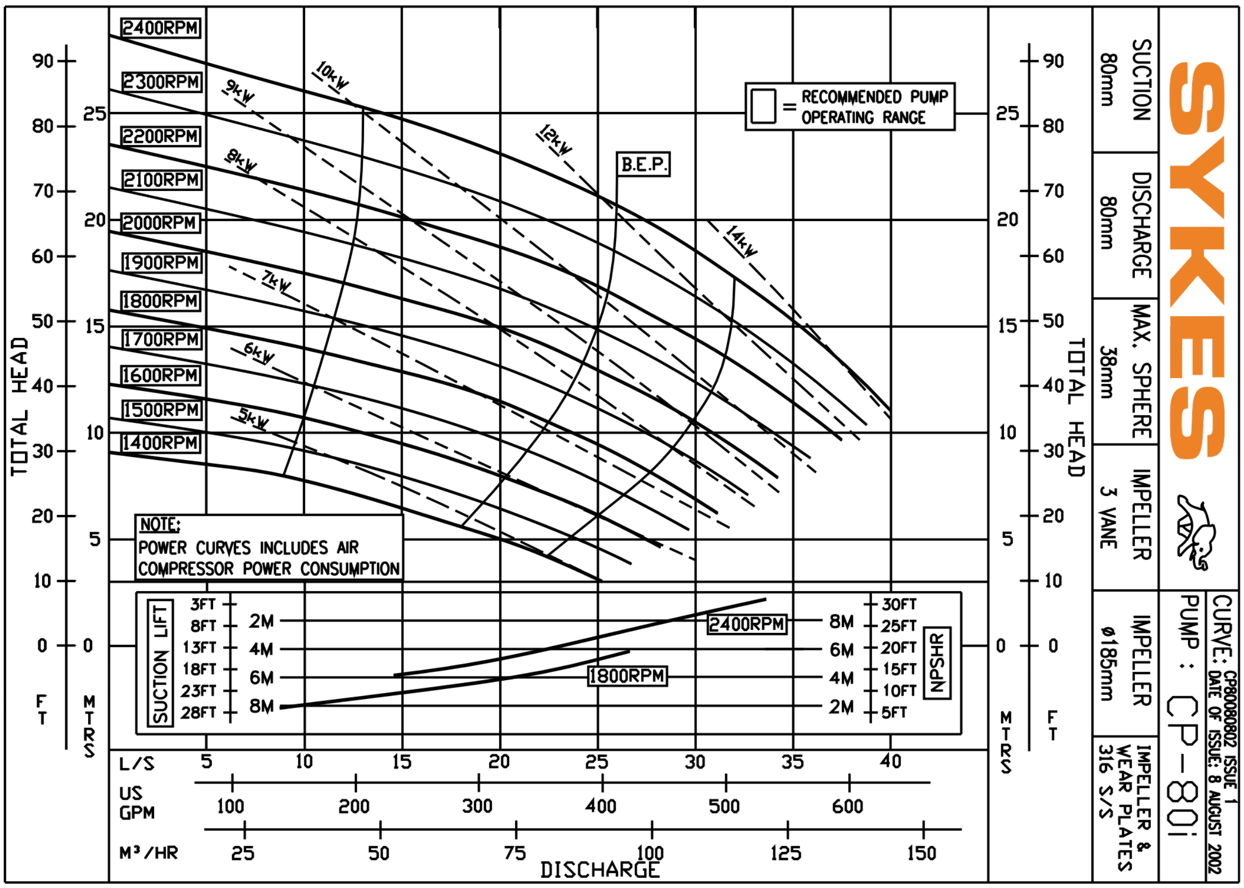
1. It is desired to pump 18 liters/sec of water from a pond to an elevation of 10 meters. If we neglect all frictional losses (say we use a really fat pipe!) what is the RPM required to do the job?

2. What is the useful mechanical work done by the pump on the fluid?

3. What is the efficiency of the pump at the operating conditions?

4. What are the total head losses (ignored above) we can tolerate before this pump is no longer recommended for the job?

5. If we are pumping hot water at 60^{0}C (vapor pressure of 150mmHg vs. that at 25^{0}C of 23.7mmHg), what is the maximum vertical distance from the level of the pond can we put the pump at, neglecting all frictional losses? (OK, a hint: mercury has a density of 13.56 times that of water…)



6. The Prandtl number is the ratio of the kinematic viscosity to the thermal diffusivity ($Pr = \nu/\alpha$). What is its approximate magnitude for a low density gas and why?

7. Give a physical description of the Reynolds stress (e.g., where does it come from, and how is it defined?).

8. Why do dimpled golf balls and fuzzy tennis balls have less drag than their smooth counterparts? One sentence, please.

9. For a shear stress of 16 dynes/cm^2 in the turbulent flow of water through a pipe, about how rough does the pipe wall have to be before it influences the flow?

10. What is the "Law of the Wall"? Where does it apply and where does it come from? Be brief!