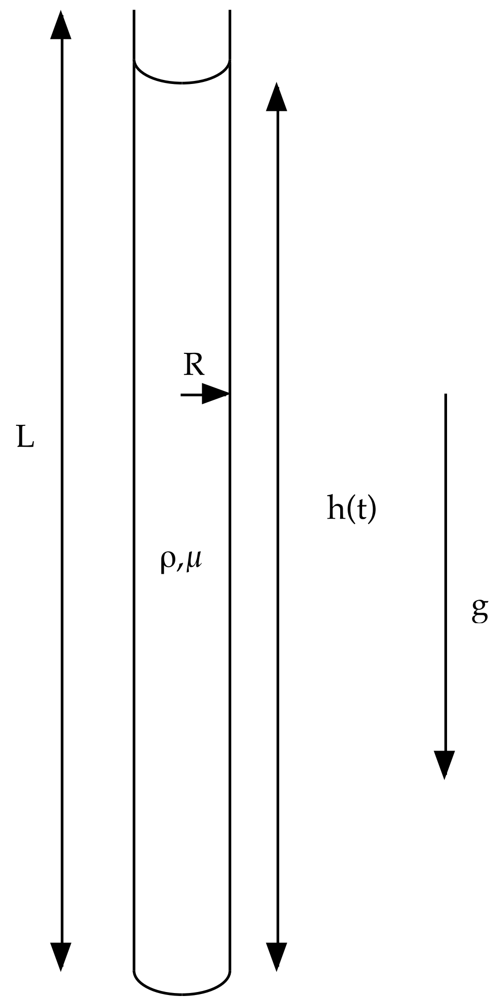
CBE 30355 **TRANSPORT PHENOMENA I** **TTh 11:00**

Second Hour Exam

Closed Books and Notes

Problem 1). (45 points) Scaling/Unidirectional Flow: In this problem we are examining the drainage of a tube of radius R and length L initially filled with a fluid of density ρ and viscosity μ as depicted below. The fluid is initially at rest (e.g., we have plugged the end of the tube with a finger!) and at t = 0 we release the fluid to drain out of the tube. We want to calculate the drainage time T_d. In the limit $R/L << 1$ we may assume unidirectional flow in the axial direction. We also ignore any surface tension effects, turbulence, film drainage issues, or other nastiness.



a. Apply the Buckingham ∏ theorem to this problem to determine how T_d depends on the various parameters of the problem. Try to pick reasonably physical dimensionless groups (there are many correct answers here, but some are obviously better than others).

b. Using the unidirectional flow approximation, write down the differential equation governing the fluid velocity (hint: unsteady, in general), keeping only the non-zero terms. Also write down the relation between the velocity and the change in height with time dh/dt of the column of fluid in the tube of length L. Write down all relevant boundary conditions and initial conditions.

c. Scale the equations for HIGH Reynolds numbers, and determine the (unknown) characteristic drainage time t_c in this limit. What boundary/initial condition(s) have to be thrown out in this limit, and what dimensionless group of parameters has to be small?

d. Solve for the actual drainage time in the high Reynolds number limit (e.g., solve the dimensionless equations obtained in part c to get the numerical value).

e. Scale the equations for LOW Reynolds numbers, and determine the new (unknown) characteristic drainage time t_c in this limit. What boundary/initial condition(s) have to be thrown out in this limit, and what dimensionless group of parameters has to be small?

f. Solve for the actual drainage time in the low Reynolds number limit (e.g., solve the dimensionless equations obtained in part e to get the numerical value).

g. If we let R = 0.5cm, L = 2 ft, and we have the properties of water, which limit is more appropriate? Be quantitative!

The following equations may be helpful (note: the vast majority of these terms may be neglected!):

$$\frac{1}{r}\frac{\partial\left(r\, v_r\right)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z}$$

$$+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho\, g_z$$

Problem 2). (15 points) Short Answer / Multiple Choice:
a. (10 pts) Briefly identify the physical mechanism described by each of the following terms:

1. $\mu\,\frac{\partial^2 v_\theta}{\partial z^2}$

2. $\frac{D\rho}{Dt}$

3. $- \rho \frac{v_\theta^2}{r}$

4. $\rho \frac{v_r v_\theta}{r}$

5. $\frac{\partial u_i}{\partial x_i} = 0$

6 - 10. (each term): $\rho \frac{\partial u_i}{\partial t} + \rho u_j \frac{\partial u_i}{\partial x_j} = - \frac{\partial p}{\partial x_i} + \mu \frac{\partial^2 u_i}{\partial x_j^2} + \rho g_i$

b. (5 pts) Multiple Choice:

1. It is proposed to study the flight dynamics of a fruit fly by building a10cm scale model. What dimensionless number do you have to keep constant to preserve dynamic similarity?
   - A. Reynolds Number
   - B. Prandtl Number
   - C. Froude Number
   - D. Weissenberg Number

2. In ship modelling we try to preserve which dimensionless number:
   - A. Reynolds Number
   - B. Prandtl Number
   - C. Froude Number
   - D. Weissenberg Number

3. Which (if any) of the following can be discontinuous at a fluid-fluid interface?
   - A. Shear rate
   - B. Heat flux
   - C. Mass flux
   - D. Velocity

4. You are struck by a desire to solve the equations of motion for the following systems. For which of the following would you use the viscous scaling? (give all that apply)
   - A. A golf ball in flight
   - B. Mixing corn syrup
   - C. The USS Enterprise
   - D. A glucose blood monitor

5. Crooke's Radiometer works because of:
   - A. The momentum of light
   - B. Thermal transpiration
   - C. Maxwell said it should
   - D. Hot gas on the black face of the vane