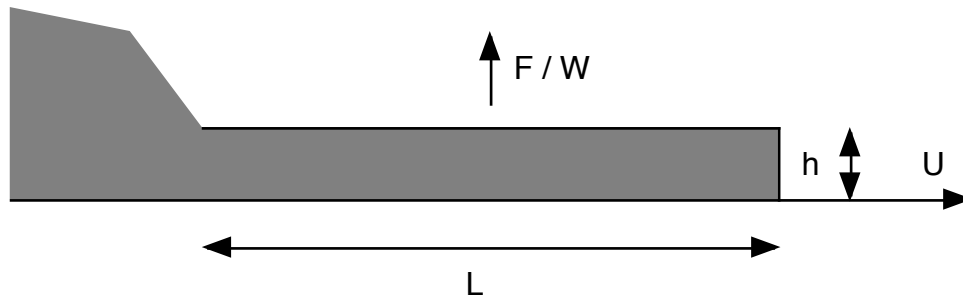


Second Hour Exam**Closed Books and Notes**

Problem 1). (20 points) Lubrication: A lubricated thrust bearing is depicted below. The lower surface moves from left to right with velocity U , dragging fluid into the gap of constant height h as depicted. The wiper at $x = L$ prevents fluid from traveling out of the bearing, resulting in a recirculation in the bearing and a pressure gradient. Here we examine the thrust per unit extension into the third dimension (e.g., F/W for this 2-D problem) produced by the bearing.



- Write down the governing equations and boundary conditions in the limit $h/L \ll 1$. Hint: one of them will be an integral relation!
- Scale the equations to determine how the force F/W scales with the parameters of the problem in the lubrication limit.
- Solve for the force.

Problem 2). (10 points) Dimensional Analysis in Ship Design: A 1:100 scale model of a ship is constructed for study of hull design in a towing tank.

- If the velocity of the model is U_m , what would be the corresponding velocity of the full scale ship?
- If the force required to move the model at this velocity is measured as F_m , what would be the corresponding required power output of the engines of the full scale ship, assuming 75% efficiency of the propellers (a reasonable value)?

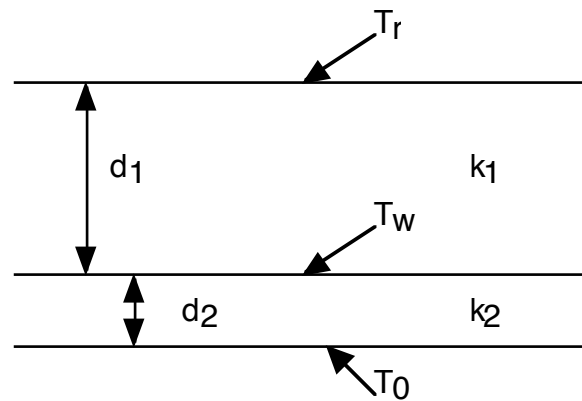
Problem 3). (10 points) Dimensional Analysis again: A porous block of material of thickness H with characteristic pore radius a is put on a puddle of liquid of viscosity μ and surface tension Γ . If the liquid wets the pores, it will be absorbed into the block over some time T .

- Using dimensional analysis, determine the independent dimensionless parameters that govern the time to fully wet the block.

b. It is observed that the wetting time scales as the square of the block thickness. Use this to strengthen your dimensional analysis.

c. If the pore radius is $10\mu\text{m}$, the block thickness is 5cm , and the fluid is water (surface tension $\Gamma = 70\text{ dynes/cm}$), what is the order of magnitude of the time to wet the block?

Problem 4). (20 points) Unidirectional Flows (or the heat transfer analog!): The waste cleanup process currently being implemented at Idaho National Lab involves a reactor where the waste is calcified for subsequent treatment. While it is supposed to be fluidized, unfortunately the “sand” produced in the reaction tends to form a solid layer on the inside of the reactor vessel, and they need to know when this occurs (it’s highly radioactive, so they can’t just look inside!). Last summer I proposed that they look at heat transfer through the vessel surface as a probe of the thickness of the solid layer, resulting in the system depicted below:



The two conductivities k_1 and k_2 , the three temperatures T_r , T_w , and T_0 , and the outer layer thickness d_2 are all known or measurable. Using this, determine what the thickness of the solid layer d_1 deposited on the wall is in terms of these values. The equations governing heat transfer in a solid are:

$$\rho C_p \frac{\partial T}{\partial t} = k \nabla^2 T \quad \tilde{q} = -k \tilde{\nabla} T$$

Note: This problem is *identical* to the unidirectional flow problem where the temperatures are equivalent to velocities, the conductivities are identical to viscosities and the heat flux q is analogous to the shear stress... Remember what has to be continuous at an interface...

Problem 5). (20 points) Short Answer:

For the five expressions below, briefly identify the corresponding physical mechanism (or equation name) **and a problem where it would play a role**:

1. $\frac{\partial \sigma_{ij}}{\partial x_j} = 0$

2. $\rho \frac{Du}{Dt} = -\nabla P$

3. $\nabla^4 \psi = 0$

4. $Q = \frac{-\pi \Delta P R^4}{8 L \mu}$

5. $\frac{D\phi}{Dt} = \frac{\partial \phi}{\partial t} + u \cdot \nabla \phi$

Briefly answer the following:

6. Estimate the Reynolds number of a 2cm diameter goldfish swimming at 20cm/s.

7. What is the difference between quasi-parallel and unidirectional flow?

8. What is the minimum dissipation theorem and where does it apply?

9. What is Stokes Law (give the equation)?

10. If two identical spheres of radius a are settling due to the force of gravity at zero Reynolds number in an infinite fluid (no boundaries), how does the vector drawn between the sphere centers change with time? Justify your answer!