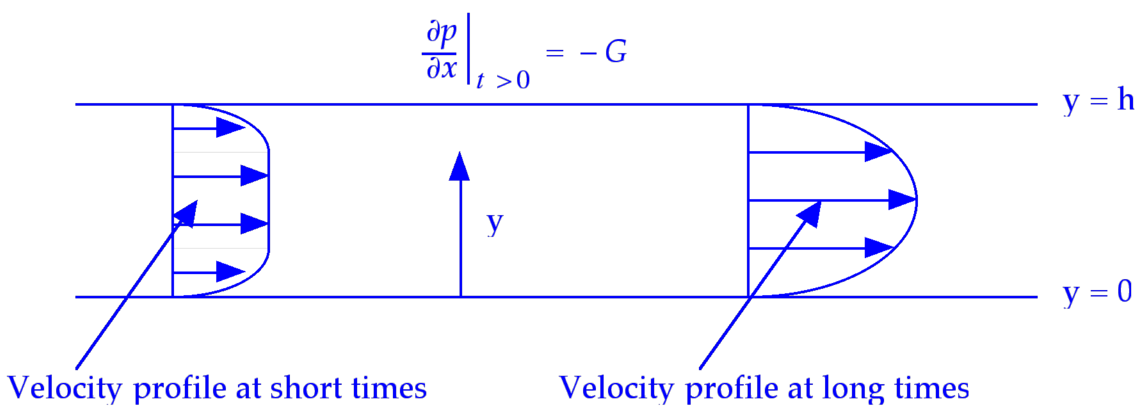
CBE 30355 Transport Phenomena I
Final Exam

December 19, 2014

Closed Books and Notes

Problem 1). (20 points) Unsteady unidirectional flow. Consider the **uni-directional time-dependent start-up flow** depicted below. Initially the fluid is at rest in a channel of width h. At time t = 0 we impose a pressure gradient dp/dx = -G (a constant) in the x-direction. This causes the fluid to accelerate, eventually reaching some asymptotic velocity distribution. In this problem we use scaling to examine all the interesting parts of the problem.



a). The starting point: Write down the governing differential equation valid for all times and associated boundary conditions, crossing out all the terms which are zero. Do not include gravity.

b). Long times: Render the equations dimensionless, using some undetermined timescale t_c for time. Show that for very large t_c (e.g., if we wait a long time), inertia no longer matters, and obtain the appropriate scaling for the asymptotic velocity. About how long do we have to wait?

c). Long times: Solve for the asymptotic velocity distribution at long times, and determine the shear stress at the lower wall (hint: the velocity distribution should be -really- familiar by now!)

d). Short times: Rescale the velocity for very short times, and show that now the viscous term becomes unimportant. How short does t_c have to be? What is the velocity scaling for short times?

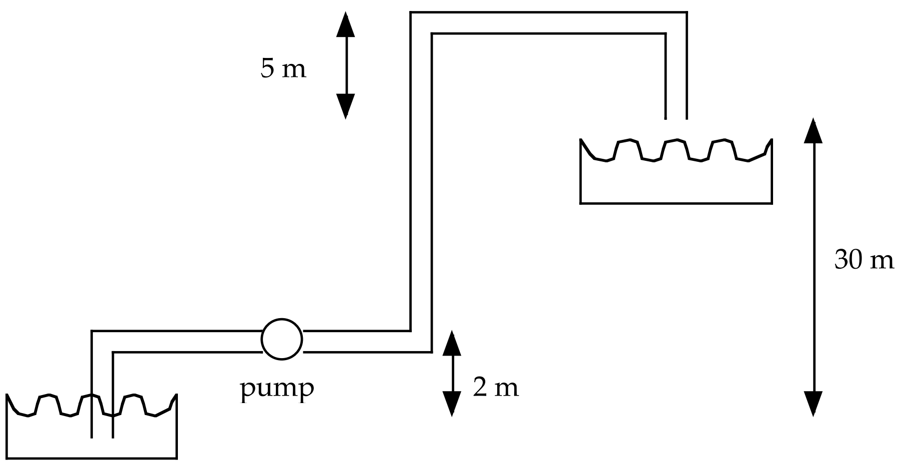
e). Short times: Solve for the now time-dependent velocity profile. (Hint: It's really simple, and you should remember what happens to the B.C.'s when you throw out the viscous term...)

f). Short times: Near each wall we will develop a boundary layer at short times. While there's one at both walls, we'll just look at the one near y = 0 for simplicity. Rescale the y coordinate and determine the boundary layer thickness as a function of t_c.

g). Short times: Using either the results of part f, or via simple affine stretching, show that the short time boundary layer problem admits a similarity solution. Give the similarity rule and the similarity variable in canonical form, and determine the time dependent wall shear stress to within some unknown multiplicative constant (e.g., the solution of the transformed differential equation that you don't have time to get).

h). All times: Using the results of (c) and (g), sketch up the wall shear stress as a function of time (rendered appropriately dimensionless) in the two asymptotic limits. Note that there is a gap between the two asymptotic solutions which must be bridged via another solution technique. It's actually fairly straightforward to get that one too, but we won't worry about that here... Do point out the domain of validity of the two solutions, however.

Problem 2). (20 points) Pumps & Pipes! Consider the piping system depicted below. You are pumping water from one reservoir to another 30 m up the hill at a rate of 10 liters/s through 5 cm ID pipe. There is 5 m of pipe leading to the pump and another 95 m leading to the second reservoir, with a total of four 90° elbows in the system. The pump you are given to use is the HH80, whose pump curve is on the next page.

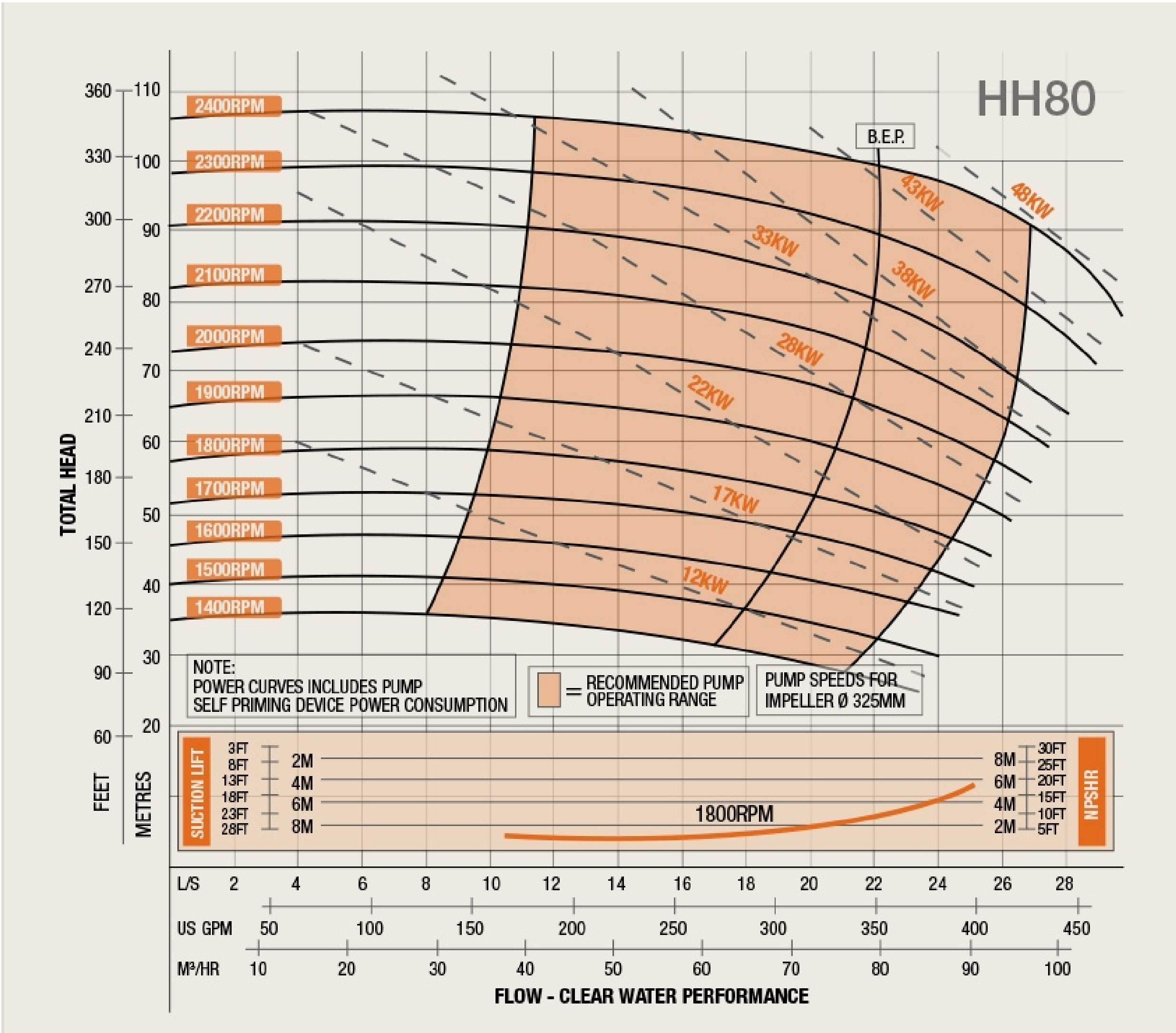


a. What is the total head the pump is required to supply (in meters of H_2O)?

b. What fraction of the energy cost of operating the pump goes into the change in elevation?

c. Will this violate NPSHR? Be quantitative!

d. Now for the main problem: Your boss isn't happy with the system, and a contractor comes by and proposes to replace the 5cm pipe with 10cm pipe, and to operate the pump half the time at twice the flow rate (e.g., 20 liters/s but 12/24 rather than continuous operation). He wants to charge \$5k for the fix. If the cost of electricity is \$0.15/kWhr, how long would it take for the reduction in operating costs to pay back the contractor's fee?



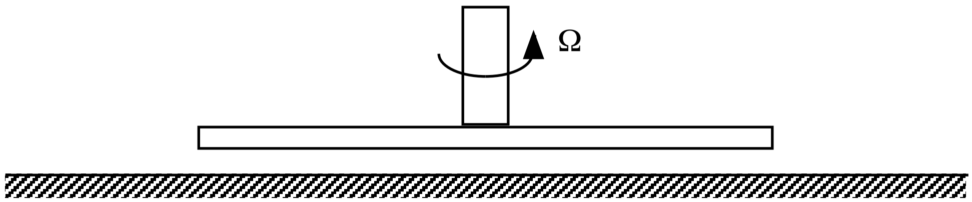
$$h_L = \frac{\langle u\rangle^2}{2g}\sum K + 4 f_f \frac{L}{D}\frac{\langle u\rangle^2}{2g}$$

$$f_f = \frac{16}{Re} \quad ; \quad \mathrm{Re} < 2100 \qquad\qquad f_f \approx \frac{0.0791}{Re^{1/4}} \quad ; \quad 3000 < \mathrm{Re} < 10^5$$

$$\frac{1}{\sqrt{f_f}} = 4.0 \log_{10}\left(\mathrm{Re}\sqrt{f_f}\right) - 0.40 \quad ; \quad \mathrm{Re} > 3000$$

Fitting	K value
sudden contraction	0.45
sudden expansion	1.0
90° elbow	0.70

Problem 3). (20 points) Lubrication/Scaling: This semester we have examined the flow resulting from a rotating disk. If the disk is far from a surface there is no axial force on it (only torque). If it is very close to a plane, however, the axial force can be very large! Here we examine this in the lubrication limit $H/R \ll 1$ where H is the separation and R is the radius, as is depicted below.



a. Calculate the θ velocity in this limit (this is simple, and you've done it before!). A classmate asserts that you also should also require $H/(\nu/\Omega)^{1/2} \ll 1$. Qualitatively describe what would happen to the velocity profile if this were violated. Be brief!

b. Write down the equation governing the radial velocity and pressure distributions in the lubrication limit. (Hint: don't forget to include the one inertial term which actually drives the radial flow!)

c. Write down all the boundary conditions which govern the radial velocity and pressure distributions. (Hint: one is an integral!)

d. Scale pressure and radial velocity, rendering the problem dimensionless. Use this to determine the scaling for the axial force on the disk.

e. To a good approximation (only off by about 10%, actually) the radial pressure gradient is just that due to the centrifugal force averaged across the gap. Using this simplifying approximation, determine a better estimate of the axial force on the disk than you can get with scaling alone (e.g., to within 10%).

The following equations *may* be helpful (particularly if you circle the dominant terms!):

$$\frac{1}{r}\frac{\partial (r\,u_r)}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} = 0$$

$$\rho\left(\frac{\partial u_z}{\partial t} + u_r\frac{\partial u_z}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta} + u_z\frac{\partial u_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2}\right] + \rho g_z$$

$$\rho\left(\frac{\partial u_\theta}{\partial t} + u_r\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta} + u_z\frac{\partial u_\theta}{\partial z} + \frac{u_r u_\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r\,u_\theta)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{\partial^2 u_\theta}{\partial z^2} + \frac{2}{r}\frac{\partial u_r}{\partial \theta}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta} + u_z\frac{\partial u_r}{\partial z} - \frac{u_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r\,u_r)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial u_r}{\partial \theta} + \frac{\partial^2 u_r}{\partial z^2}\right] + \rho g_r$$

Problem 4). (20 points) Short Answer:

1. What is the mathematical representation of the material derivative, and what does it physically represent?

2. What is boundary layer separation, and why does it matter?

3. What is a yield stress, and name a common fluid that possesses one.

4. Give a physical description of the Reynolds stress (e.g., where does it come from, and how is it defined?).

5. What is the friction velocity (in terms of stress, etc.) and where is it used?

6. How is the Prandtl number defined, and what does it represent?

7. How is the drag coefficient defined at high Re?

8. What is the difference between quasi-parallel flow and uni-directional flow? Give a (brief!) example of each.

9. What are Taylor-Couette vortices, and why do they occur?

10. What is the capillary number Ca? (How is it defined, and what is it good for?)