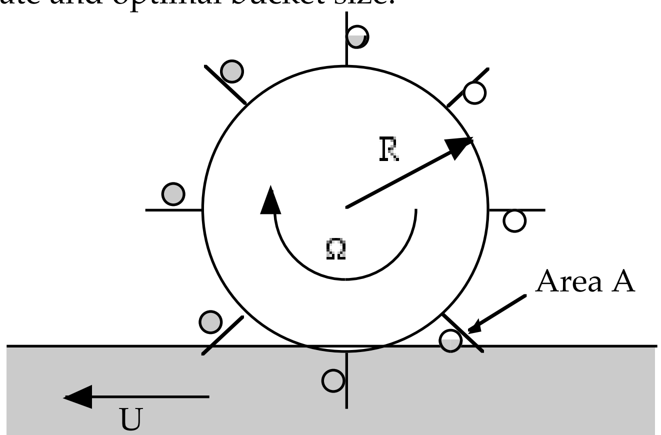
CBE 30355 TRANSPORT PHENOMENA I

First Hour Exam
10/5/10

This test is closed books and closed notes

Problem 1. (20 pts) Off I 26 in Orangeburg, SC (on the route to Hilton Head) is Edisto Memorial Gardens. Probably the most photographed feature of the gardens is a 70 year old waterwheel that was once used to pump water from the slow moving Edisto blackwater river to a nearby pond. Such undershot waterwheels were often used to irrigate rice paddies, for example. The operation is simple: the flowing water applies a force to the vanes of area A, causing the wheel to turn. As each vane enters the water a bucket fills with water and then empties at the top of the arc. Your mission is to determine the flow rate and optimal bucket size.

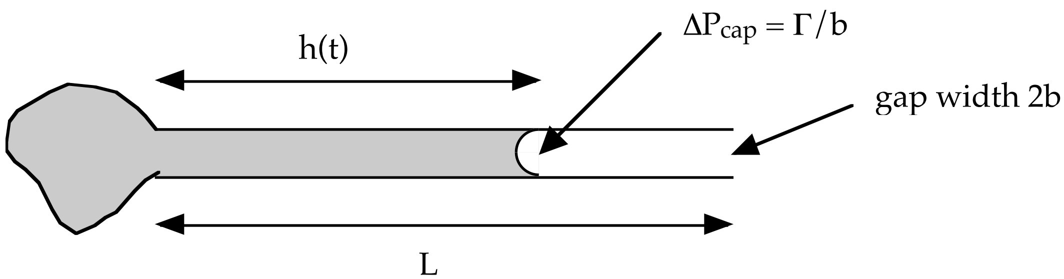


a. This problem isn't as "clean" mathematically as the idealized Pelton wheel solved in class, but the physical principle is the same. It is easiest to work with the total volume of all the buckets V_T, as then the number of vanes doesn't play a large role (assuming there are a lot of them, and one of area A is perpendicular to the stream at all times). Making any further assumptions necessary (but state them clearly so I can see how you get your answer!), develop an approximate equation for the flow rate Q of this pump.

b. What is the optimum value of V_T for a fixed river speed U? What is the maximum value of V_T for which the equation obtained in part a is valid?

Hint: The torque exerted on the wheel due to the filled buckets on one side (total filled volume $V_T/2$) is approximately $\rho\, g\, V_T\, R\, /\, \pi$.

Problem 2. (20 pts) Earlier this semester we briefly looked at capillary pressure effects, whose magnitudes are characterized by the surface tension Γ. One way fluid (e.g., blood) is drawn into a microfluidic channel for analysis is using capillary wetting, the same effect that causes water to wick into a paper towel. In this problem we examine the capillary wetting of fluid in a narrow 2-D channel. Consider the system depicted below. Fluid enters the thin gap of width 2b. If the fluid wets the surfaces of the channel (basically, if a drop of the fluid would spread out over the material composing the channel surfaces), then the curvature of the interface will cause a capillary pressure drop between the fluid and the air of $\Delta p_{cap} = \Gamma/b$. Because the pressure in the fluid behind the meniscus is now *less* than that in the reservoir, it sucks fluid into the gap from the reservoir. After some time T_f the fluid fills the channel (and thus would reach the analysis section). It is this time we wish to determine.

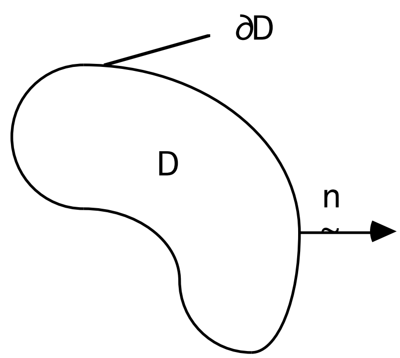


a. For very thin channels inertial (including the term arising from accumulation of momentum!), gravity, and end effects will be negligible and we will get unidirectional flow. If, at some instant in time, the fluid has filled the channel to a length h, calculate the velocity distribution and flow rate.

b. Now for the transient problem. Using the result of part a, develop an equation for how h evolves in time (initially h=0, e.g., the channel is empty). Use this to determine the fill time T_f where h = L.

Problem 3. (20 pts)

a. Starting from the arbitrary stationary control volume depicted below, derive the continuity equation for a *compressible* fluid.



b. Write down the Navier-Stokes equations for an incompressible fluid. There are five terms in these equations. Provide a *brief* physical interpretation of each term and where it comes from.

Problem 4. (20 pts) Short Answer / Index notation / Additional Readings

1. (1 pt) Flow visualization using the time-lapsed photography of a tracer particle technique reveals which of the below for **unsteady** flows?
   A. Pathlines
   B. Streaklines
   C. Streamlines
   D. All of the above

2. (1 pt) Flow visualization using the dye release technique (dye ejected from a fixed array of nozzles) reveals which of the below for **steady** flows?
   A. Pathlines
   B. Streaklines
   C. Streamlines
   D. All of the above

3. (2 pts) In 1919 Boston was inundated by a flood of what fluid? What material property played a critical role in *causing* the disaster?

4. (2 pts) Provide two physical interpretations of the vector $\rho \underset{\sim}{u}$.

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6. (2 pts) Match up the kinematic viscosities of the following materials:

1. Water	A. 650 cSt
2. Air	B. 17.0 cSt
3. Glycerine	C. 1.0 cSt
4. Mercury	D. 0.118 cSt

7. (2 pts) Write down the continuity equation for an incompressible fluid using index notation.

8. (2 pts) Write down the index notation representation of the rate of strain tensor for a velocity field u_i.

9. (2 pts) Using index notation, decompose the general matrix A_{ij} into the sum of symmetric and antisymmetric parts.

10. (2 pts) The vorticity is defined as the curl of the velocity. Write down this defining relation using index notation.

11. (2 pts) What is the most general second order physical tensor specified by a single director p_i?