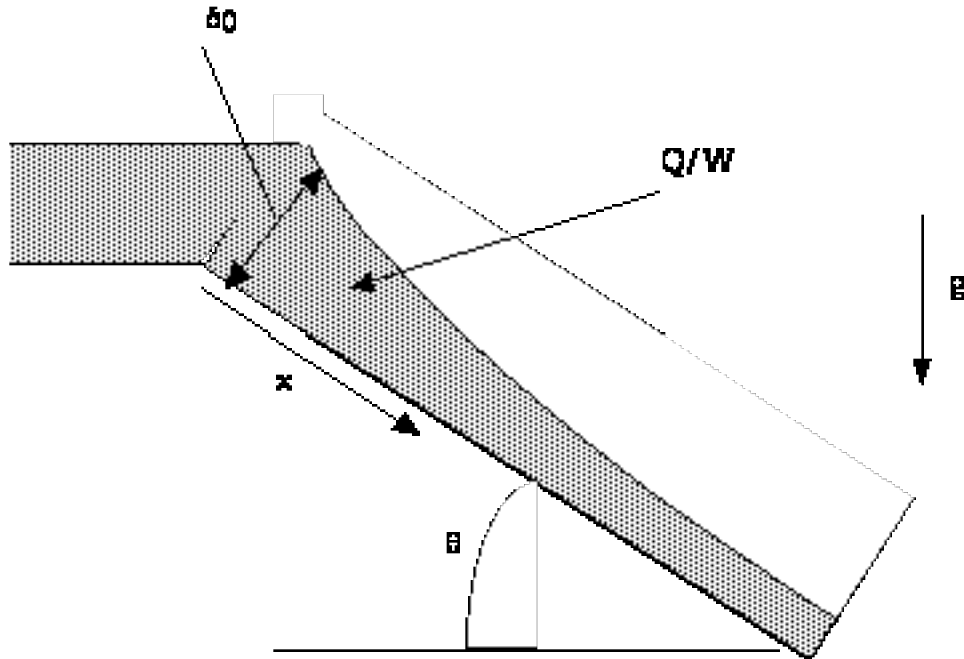**Second Hour Exam**

Closed Books and Notes

Problem 1). (30 points) High Reynolds Number Flows: For homework you analyzed the problem of a fluid flowing down an incline, such as depicted below, for low Re. In this problem we look at the opposite high Re limit.


a. (20pts) Ignoring all frictional losses, calculate the depth of the fluid as a function of distance down the incline. The initial depth is δ_0 and the flow rate per unit width is Q/W.

b. (10pts) The fluid will keep getting thinner as it accelerates down the incline. Eventually, however, frictional losses due to turbulence will come into play, and the fluid will reach some asymptotic thickness. We want to estimate this thickness. The effect of turbulence can be modeled as an "eddy viscosity" analogous to the molecular viscosity in the equations of motion, only now the viscosity will be a function of both the depth of the layer and the velocity. For this problem the eddy viscosity can be estimated as:

$$\mu^{(t)} \approx \lambda \rho \, \delta^2 \left| \frac{u}{\delta} \right|$$

The parameter λ is just a dimensionless constant, with a value of around 0.1 or so. Replacing the molecular viscosity in the equation of motion with this term, use scaling analysis to estimate the asymptotic depth of the fluid layer. (You don't have to -solve-it, just determine how it scales with the various physical parameters).

Problem 2). (30 points) Creeping Flows: In this problem we examine the lid driven cavity flow depicted below. The top plate is dragged to the right (positive x direction) with velocity U, while the bottom is fixed and the side walls produce a pressure gradient and resulting backflow to the left (negative x direction). The flow may be taken to be at zero Re. Your job is to figure out what is going on.
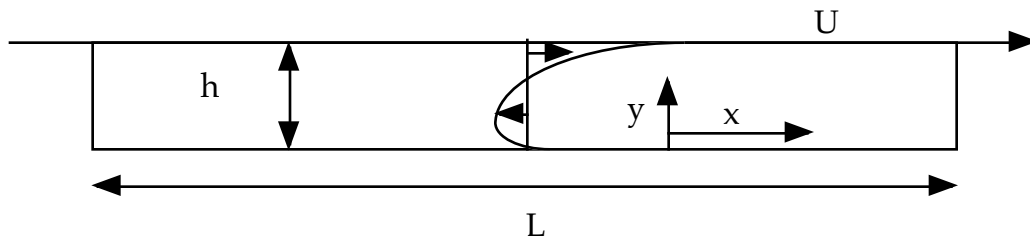
a. (20pts) In the limit $h/L \ll 1$ (thin gaps), determine the velocity profile and pressure gradient far from the ends. This means that you ignore the end regions where the flow field gets nasty.

b. (10pts) What (hint: single!) equation governs the flow when the aspect ratio is O(1)? (e.g., h/L is no longer small). What are the appropriate boundary conditions for this equation?

Problem 3. (30 points) Short Answer

a. (10 pts) For each of the following equations, briefly list the most important conditions under which it is valid, (e.g., what flows is it good for?).

1. $\nabla^4 \psi = 0$

2. $\dfrac{\partial \sigma_{ij}}{\partial x_j} = 0$

3. $p + \dfrac{1}{2}\rho u^2 + \rho g h = Cst$

4. $\dfrac{\partial u_i}{\partial x_i} = 0$

5. $\tilde{u} = -\tilde{\nabla} \phi$

b. (10 pts) Briefly identify the physical mechanism described by each of the following terms:

1. $\mu \dfrac{\partial^2 u_x}{\partial y^2}$

2. $\tilde{\nabla} \times \tilde{u}$

3-5. (each term): $\dfrac{\partial T}{\partial t} + \tilde{u} \cdot \tilde{\nabla} T = \alpha \nabla^2 T$

c. (10 pts) Multiple Choice:

1. In the study of insect flight, the boffins of Berkeley looked at the behavior of the 10cm scale model of a fruit fly in which fluid:
    A. Mercury
    B. Water
    C. Mineral Oil
    D. Air
    E. Liquid Nitrogen

2). Which (if any) of the following can be discontinuous at a fluid-fluid interface?
    A. Velocity
    B. Shear stress
    C. Heat flux
    D. Mass flux
    E. Shear rate

3). Who determined the yield of the H-bomb tests by dimensional analysis?
    A. Buckingham
    B. Reynolds
    C. Maxwell
    D. Taylor
    E. Batchelor

4). Flows on a LabChip are often (give all that apply):
    A. Low Reynolds Number
    B. High Reynolds Number
    C. Electrokinetic
    D. Low Prandtl Number

5). Order the drag of the following objects under creeping flow conditions
    A. A cube 2 cm on a side
    B. A sphere 1.75 cm in radius
    C. A sphere 2 cm in diameter