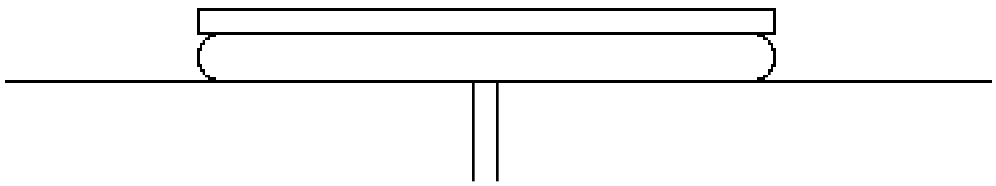
CBE 30355 **TRANSPORT PHENOMENA I** **TTh 11:00**

Second Hour Exam

Closed Books and Notes

Problem 1). (30 points) Scaling/Lubrication Theory: Consider a disk of radius R located a distance H above a plane as depicted below. Fluid of viscosity μ is pumped into the center of this geometry at a rate Q (volume/time), and flows outward radially. This flow results in a pressure gradient, and hence develops a high pressure underneath the disk, applying an upward force on it. In this problem, you are to determine this force. You may simplify the problem by looking at the lubrication limit $H/R \ll 1$.



a. Using a mass balance, determine the characteristic magnitude of the radial velocity.

b. Via scaling analysis of the equations of motion, determine the characteristic magnitude of the pressure in the lubrication limit.

c. How does the magnitude of the force depend on the parameters of the problem? (e.g., get the force to within an unknown (but hopefully order one) constant)

d. Solve for the velocity distribution, pressure distribution, and force to get this constant.

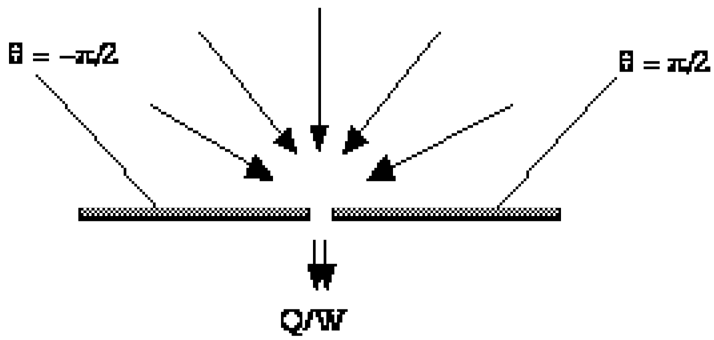
The following equations may be helpful (note: the vast majority of these terms may be neglected!):

$$\frac{1}{r}\frac{\partial (r\, v_r)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0$$

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r}$$
$$+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\, v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right] + \rho\, g_r$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z}$$
$$+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho\, g_z$$

Problem 2). (30 points) Two Dimensional Flows: Consider the two-dimensional geometry depicted below. A very viscous fluid is draining out of the bottom of a tank through a narrow slit at a rate Q/W per unit extension into the paper.



a. Set up the problem as completely as possible, with all boundary conditions clearly determined.

b. Using the boundary conditions as a guide, determine the form of the streamfunction, and develop the corresponding ODE and boundary conditions. Hint: remember the relationship between the net flow rate through any curve connecting two streamlines, and the value of the streamfunction on the streamlines!

c. Solve for the streamfunction. Make maximum use of symmetry!

Remember that in the cylindrical geometry we have:

$$v_\theta = \frac{\partial \psi}{\partial r} \quad , \quad v_r = -\frac{1}{r}\frac{\partial \psi}{\partial \theta}$$

and

$$\nabla^2 \psi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta^2}$$

Problem 3). (10 points) Short Answer: Identify the following equations and state under what conditions they are valid:

a. $\nabla^4 \psi = 0$

b. $\dfrac{\partial u_i}{\partial x_i} = 0$

c. $\dfrac{\partial \sigma_{ij}}{\partial x_j} = 0$

d $E^4 \psi = 0$

e. $\tilde{u} = -\tilde{\nabla} \phi$

Problem 4). (10 pts) Dimensional Analysis: You are assigned the task of estimating the drag on a 300 meter long aircraft carrier moving at 20 m/s (roughly 40 knots, or 40 nautical miles per hour). You choose to do this using a towing tank with a 1 meter long model.

a. The dimensionless drag depends on two key dimensionless parameters: what are they, and what is their approximate magnitude for the full-sized ship? (Hint: If you can't remember them, you can easily get them by appropriate scaling of the Navier-Stokes equations!)

b. Using the concept of approximate (rather than strict) dynamic similarity, determine the velocity at which the model should be towed to best approximate the flow pattern of the full-sized carrier. What are the values of the important dimensionless parameters for the model?

c. What is the ratio of the drag on the carrier to that measured for the model?