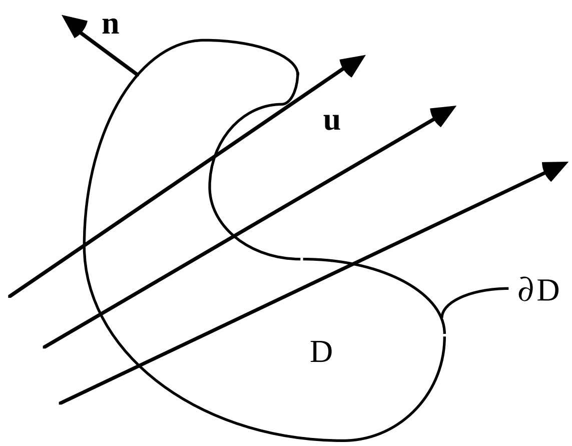
CHEG 355 Transport Phenomena I
Final Exam

December 18, 2002

Closed Books and Notes

Problem 1). (20 points) Plane Poiseuille Flow: A problem which is currently being investigated in bioengineering laboratories is the phenomenon of cell adhesion to surfaces in the presence of hydrodynamic stresses. This is very important in the design of biocompatible materials, for example. To study this, a researcher has built a rectangular flow cell which is 100μm deep, 2mm wide, and 2cm long. The objective is to have a wall shear stress (e.g., stress at the lower wall - the 2mm x 2cm surface - where cell adhesion is being studied) of 100 dyne/cm^2. If the working fluid has the same viscosity as water, what should the flow rate of the pump supplying the fluid be? Make any simplifications you need to, but provide a **brief** justification.

Problem 2). (20 points) Conservation of Momentum: Given that the momentum flux is given by the stress tensor $\boldsymbol{\sigma}$ and the acceleration due to gravity $\mathbf{g}$ acts as a source of momentum, derive the Cauchy Stress Equations (e.g., leave the momentum equations in terms of $\boldsymbol{\sigma}$).

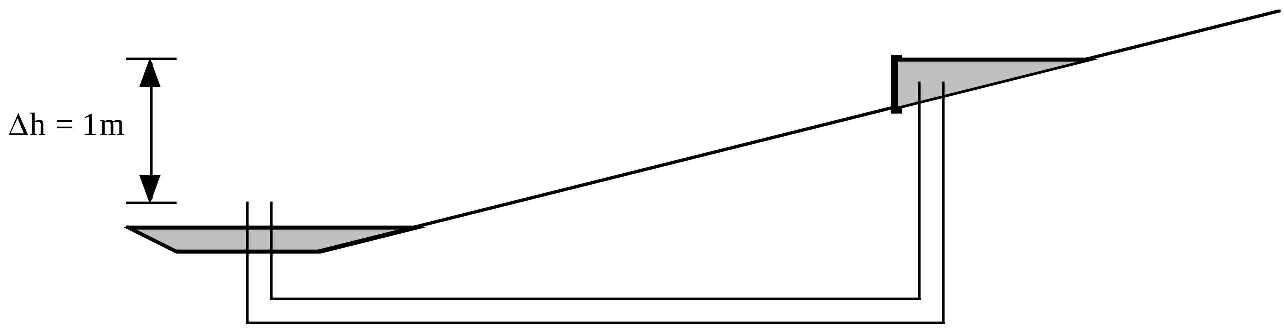


Problem 3. (20 points) A Simple Fountain: The ancient Greeks and Romans (as well as many later civilizations, including the famous fountains of Versailles!) often operated fountains by the simple expedient of hydrostatic pressure produced by a reservoir fed by springs. In this case we will examine a very simple design for a small garden fountain driven by a reservoir in a creek. Consider the pipe system and reservoir depicted below. All pipes are smooth 2cm diameter and the total length of pipe is 20m. The reservoir is located 1m above the pool where we are installing the fountain.

a. Neglecting all losses, what is the maximum height the fountain could reach (think simple here!)?

b. Accounting for frictional losses, what height does it actually get to? (Hint: Not very impressive!)

c. What is the cheapest way we could get a fountain height much closer to the theoretical maximum?



$$h_L = \frac{\langle u \rangle^2}{2g} \sum K + 4 f_f \frac{L}{D} \frac{\langle u \rangle^2}{2g}$$

$$f_f = \frac{16}{Re} \;;\; \text{Re} < 2100 \qquad f_f \approx \frac{0.0791}{Re^{1/4}} \;;\; 3000 < \text{Re} < 10^5$$

$$\frac{1}{\sqrt{f_f}} = 4.0 \log_{10}(Re\, f_f) - 0.40 \;;\; \text{Re} > 3000$$

Fitting	K value
sudden contraction	0.45
sudden expansion	1.0
90° elbow	0.90

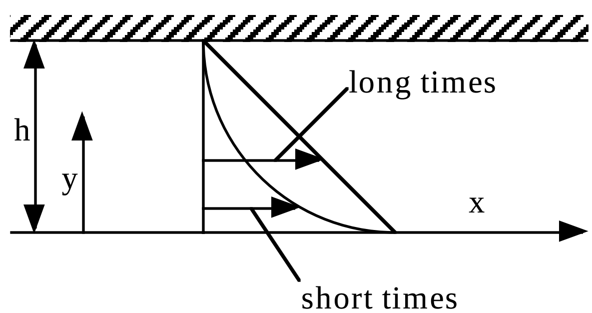
Problem 4. (20 points) Uni-directional Startup Flows. Consider the channel depicted below. For all times $t < 0$ the lower plate and fluid are at rest. At $t = 0$ the lower plate is accelerated such that the shear stress at the wall is a constant τ_w independent of time. The flow may be regarded to be unidirectional in the x-direction.

a. Determine the differential equation and boundary conditions governing this problem.

b. At very long times the flow reaches steady-state. Using t_c as this time scale and h as the length scale, how large does t_c have to be for the flow to become steady? (Hint: use scaling analysis and figure out what dimensionless group has to be large or small for the appropriate term to be thrown away). What is the velocity profile in this limit?

c. For very short times a boundary layer develops near the accelerating plate. Using t_c as this time scale, determine the characteristic thickness of this boundary layer (e.g., the new length scale) and the characteristic velocity of the lower plate.

d. Using the technique of coordinate stretching, show that this short-time limit admits a self-similar solution and obtain the similarity rule and variable in canonical form. Determine the velocity of the lower plate as a function of time to within an unknown O(1) multiplicative constant (e.g., the solution of the similarity problem). Approximately when will this solution no longer be valid?



Problem 5. (20 points) Pump Curves / Additional Readings:

The first five questions refer to the pump curve on the next page:

1. It is desired to pump 25 liters/sec from a pond to an elevation of 15 meters. If we neglect all frictional losses (say we use a really fat pipe!) is the pump CP80 recommended for the job?

2. What is the RPM required to do the job?

3. What is the work done <u>by</u> the pump <u>on</u> the fluid?

4. What is the efficiency of the pump at the operating conditions?

5. It is proposed that the pump be placed 5m up the hill from the level of the pond. Again neglecting frictional losses, will this work? (Note: 1atm ≈ 10.3 m water)

6. At high Re, drag principally results from:

A. Potential Flow
B. Boundary Layer Separation
C. Skin Friction
D. Turbulence

7. Why do dimpled golf balls and fuzzy tennis balls have less drag than their smooth counterparts? One sentence, please.

8. Why did the Tacoma Narrows Bridge collapse? One sentence, please.

9. Ensuring that the NPSHR is met for a pump installation will help prevent:

A. Cavitation in the pump
B. Damage to pipes and fittings due to vibrations
C. Reduced pump performance
D. All of the above

10. Write down the Navier-Stokes equations in index notation.