CBE 30355 Transport Phenomena I
Final Exam

December 12, 2006

Closed Books and Notes

Problem 1. (20 points) Thermal boundary layers. Consider the system depicted below. The fluid velocity is just unidirectional simple shear flow in the x-direction. The fluid enters with a temperature of zero (e.g., we've already subtracted off some reference temperature), and gets heated by a wall heat flux that is **linearly increasing** as we travel down the plate, e.g., $\mathbf{q_w = \lambda\, x}$ where λ is some constant. The thermal energy equation is given by:

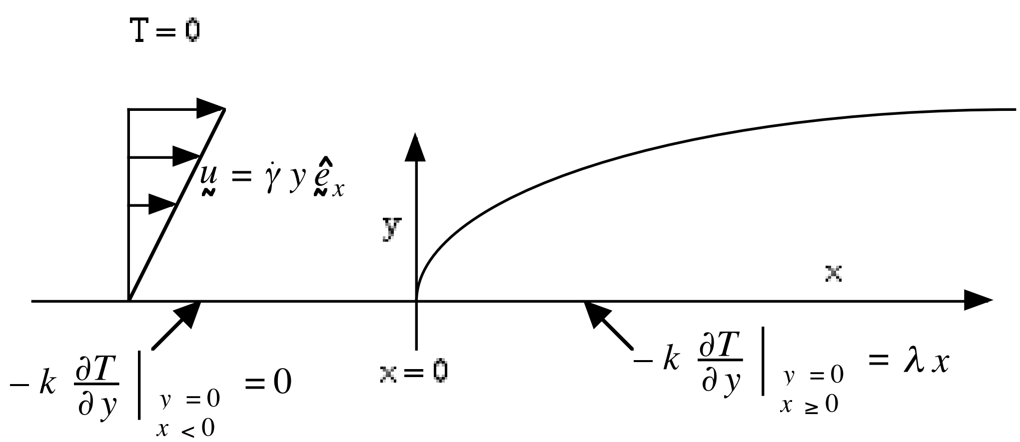
$$\frac{\partial T}{\partial t} + \underset{\sim}{u} \cdot \underset{\sim}{\nabla}\, T = \alpha \nabla^2 T$$

with boundary conditions:

$$-k \left.\frac{\partial T}{\partial y}\right|_{\substack{y=0 \\ x \ge 0}} = \lambda x \quad , \quad -k \left.\frac{\partial T}{\partial y}\right|_{\substack{y=0 \\ x \le 0}} = 0 \quad , \quad T\big|_{y \to \infty} = 0$$

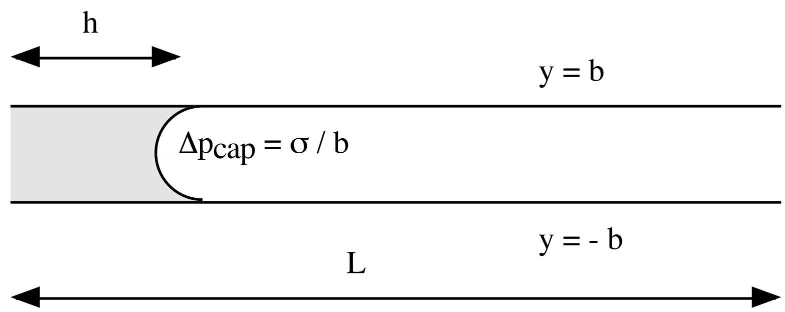
a). Render the governing equation and boundary conditions dimensionless using a length scale L in the x-direction, and determine the conditions under which we can expect a thin boundary layer in the y-direction (e.g., how big does L have to be?). You may assume steady-state, with no variation in the z-direction.

b). Show that the thermal boundary layer equations yield a self-similar solution, obtaining the similarity rule and similarity variable in canonical form. Using this, determine the temperature at the plate as a function of x to within some undetermined multiplicative constant. Note that you don't have to get the transformed ODE or solve it to do this!



Problem 2. (20 pts) Analysis of blood chemistry is now being done by putting a tiny drop of blood onto the entrance of a microchannel, and the blood is drawn into the channel via capillary action. This is the same effect that allows fuel to be drawn up the wick of a lamp, for example. In this problem we examine the length of time necessary for the channel to fill.

Consider the channel of depth 2 b and length L given below. The curvature of the fluid interface causes the pressure behind the meniscus to be less than atmospheric pressure by an amount $\Delta p_{cap} = \sigma/b$. Because the channel dimensions are very small, inertial effects are negligible and the force due to the capillary pressure is balanced by viscous effects only.



a. Using this information, determine the equations governing the velocity distribution in the channel, and the length h of the column of fluid in the channel as a function of time. Make any approximations or simplifications necessary to get a reasonable problem. (Hint: think of plane-Poiseuille flow, flow rates, and mass balances)

b. Using scaling analysis, render the problem dimensionless and determine the dependence of the fill time on all the parameters of the problem.

c. Solve for the numerical value of the fill time with the initial condition h(0) = 0 (e.g., the channel is initially empty).

(As a side note, the analysis you do above works very well provided the channel depth is greater than around 50μm. For microchannels thinner than this value, some fascinating migration effects of the red blood cells become significant, and you have to do a much more sophisticated analysis of the problem!)

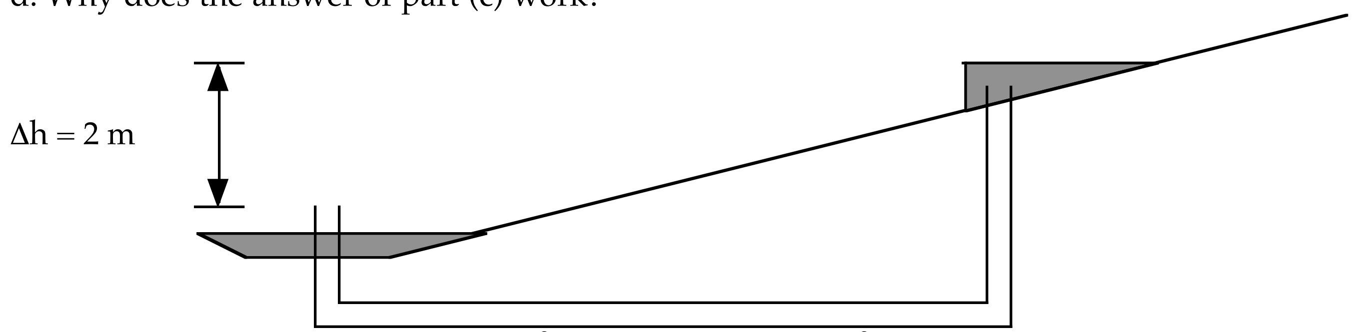
Problem 3. (20 points) A Simple Fountain: The ancient Greeks and Romans (as well as many later civilizations, including the famous fountains of Versailles!) often operated fountains by the simple expedient of hydrostatic pressure produced by a reservoir fed by springs. In this case we will examine a very simple design for a small garden fountain driven by a reservoir in a creek. Consider the pipe system and reservoir depicted below. All pipes are smooth 5 cm ID and the total length of pipe is 40 m. The reservoir is located 2 m above the pool where we are installing the fountain.

a. Neglecting all losses, what is the maximum height the fountain could reach (think simple here!)?

b. Accounting for frictional losses, what height does it actually get to?

c. What is the cheapest way (spending no more than $2 at Menards) we could get a fountain height much closer to the theoretical maximum?

d. Why does the answer of part (c) work?



$$h_L = \frac{\langle u \rangle^2}{2g} \Sigma K + 4 \; f_f \frac{L}{D} \frac{\langle u \rangle^2}{2g}$$

$$f_f = \frac{16}{Re} \; ; \; \mathrm{Re} < 2100 \qquad f_f \approx \frac{0.0791}{Re^{1/4}} \; ; \; 3000 < \mathrm{Re} < 10^5$$

$$\frac{1}{\sqrt{f_f}} = 4.0 \log_{10}\left(\mathrm{Re}\sqrt{f_f}\right) - 0.40 \; ; \; \mathrm{Re} > 3000$$

Fitting	K value
sudden contraction	0.45
sudden expansion	1.0
90° elbow	0.90

Problem 4. (30 points) Pump Curves / Additional Readings / Short Answer:

The first seven questions refer to the pump curve on the last page:

1. It is desired to pump 150 liters/sec from a pond to an elevation of 10 meters. If we neglect all frictional losses (say we use a really fat pipe!) is the pump CP200 recommended for the job?

2. What is the RPM required to do the job?

3. What is the work done <u>by</u> the pump <u>on</u> the fluid per unit time?

4. What is the efficiency of the pump at the operating conditions?

5. How far up the hill from the level of the pond can we put the pump? (Again, neglect frictional losses) (Note: 1atm ≈ 10.3 m water)

6. Frictional losses always add to the required head. What additional head losses can we tolerate before the pump is unable to achieve the required flow rate?

7. It is proposed to use a 10cm diameter pipe for this system. Is this reasonable for this flow rate? Briefly (and quantitatively) justify your answer!

8. The turbulent friction velocity is a characteristic velocity created from a shear stress and fluid properties. What is it in terms of these parameters?

9. The turbulent viscous length scale is a characteristic length created from a shear stress and fluid properties. What is it in terms of these parameters?

10. What is the approximate thickness of the viscous sublayer in plus units?

11. For a shear stress of 25 dynes/cm^2 in the flow of water through a pipe, about how rough does the pipe wall have to be before it influences the flow?

12. Give a physical description of the Reynolds stress (e.g., where does it come from?).

13. What does NPSHR stand for, and why should we care?

14. Sketch and briefly explain the principle behind a venturi flow meter.

15. Sketch and briefly explain the principle behind an orifice flow meter.

SYKES PUMPS

CURVE: CP2000108 ISSUE 3
DATE OF ISSUE 9 AUGUST 2002

PUMP : CP–200

SUCTION	DISCHARGE	MAX. SPHERE	IMPELLER	IMPELLER	IMPELLER & WEAR PLATES
200mm	200mm	80mm	3 VANE	ø295mm	316 S/S