CBE 30355 Transport Phenomena I
Final Exam

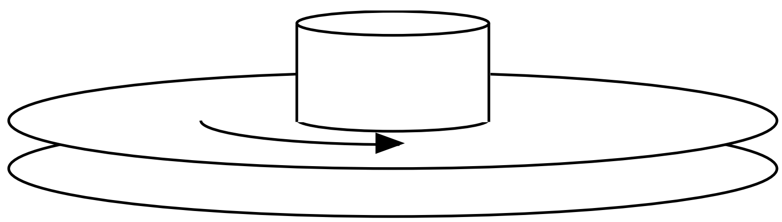
December 17, 2007

Closed Books and Notes

Problem 1. (20 points) Parallel-Plate Flow: A controlled stress rheometer is depicted below. It works by imposing some specified torque to the upper plate, and then measuring the resulting angular velocity. The viscosity of the fluid between the rotating upper plate and stationary lower plate is proportional to the ratio of the torque and the resulting angular velocity.

a). Solve for the relationship between the torque T and the angular velocity Ω in terms of the gap width h, the plate radius R and the fluid viscosity μ. For this system it is appropriate to assume low Reynolds number flow, greatly simplifying the problem!

b). Experimentally, the measured gap width often has some error in it, principally due to errors in the "zero" as you examined in a problem earlier this semester. If the actual gap width is $h = h_{meas} + \Delta h$, explicitly show how you can determine the correct viscosity by measuring Ω for two different measured gap widths h_1 and h_2 at the same torque T.



You may find the following equations helpful:

$$\frac{1}{r}\frac{\partial (r\, v_r)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta}$$

$$+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\, v_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2}\right] + \rho\, g_\theta$$

$$\tau_{z\theta} = \tau_{\theta z} = \mu\left[\frac{\partial v_\theta}{\partial z} + \frac{1}{r}\frac{\partial v_z}{\partial \theta}\right]$$

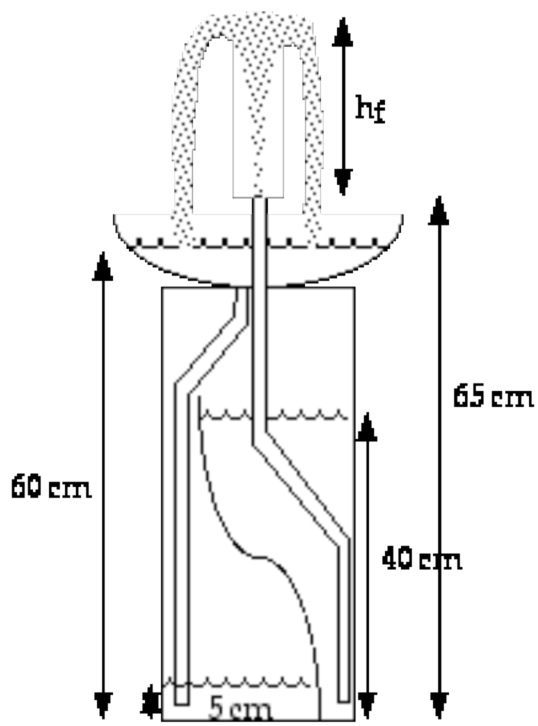
$$\underset{\sim}{T} = \int_{\partial D} \underset{\sim}{r} \times \underset{\sim}{f}\, dA$$

Problem 2. (20 points) A Variant on Hero's Fountain.

a). Neglecting all frictional losses, what is the height h_f of the fountain water jet?

b). Modify your answer by accounting for the head losses in the pipes and fittings. Correlations for friction factors in pipes and fittings are given below. You may take the total length of pipe to be 120 cm of 0.5 cm ID tubing. You will probably need to do a couple of iterations to get the friction factor right.

c). The result in part b leads to a rather unimpressive fountain. What is the most practical way of fixing this, and why does it work?



$$h_L = \frac{\langle u \rangle^2}{2g} \sum K + 4 \; f_f \frac{L}{D} \frac{\langle u \rangle^2}{2g}$$

$$f_f = \frac{16}{Re} \;\; ; \; \text{Re} < 2100 \qquad\qquad f_f \approx \frac{0.0791}{Re^{1/4}} \;\; ; \; 3000 < \text{Re} < 10^5$$

$$\frac{1}{\sqrt{f_f}} = 4.0 \log_{10}\left(\text{Re}\sqrt{f_f}\right) - 0.40 \;\; ; \; \text{Re} > 3000$$

Fitting	K value
sudden contraction	0.45
sudden expansion	1.0
45° elbow	0.35

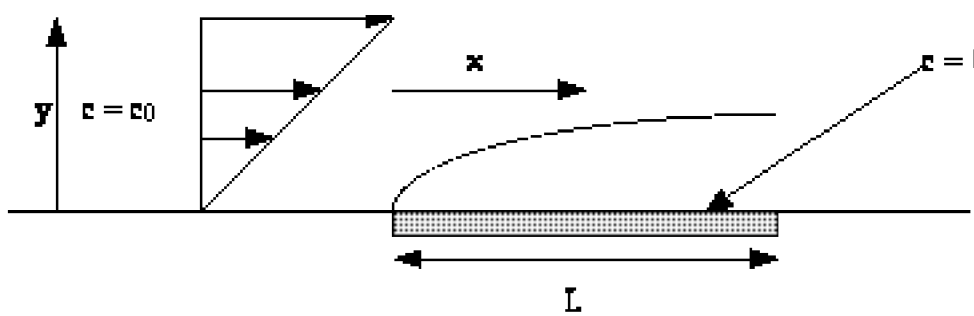
Problem 3). (20 points) Transient Boundary Layer Scaling. You are asked to design an electrochemical probe capable of measuring both concentration and wall shear stress next to a wall. The idea is that fluid flows along the surface of the wall containing some concentration c_0, and then at the surface of the probe we electrochemically reduce the concentration to zero over the whole length L. Initially, you develop a transient boundary layer independent of x, and then at long times you develop a steady, x-dependent boundary layer independent of t. The total reaction rate per unit width (extension into the paper) Q/W can thus be used to determine both c_0 and τ_w by examining these two limits.

a). Scale the equations and boundary conditions at short time scales t_c to determine how the transient boundary layer thickness δ_t and Q/W depend on the parameters of the problem.

b). Scale the equations and boundary conditions at long times to determine how the steady boundary layer thickness δ_s and Q/W depend on the parameters of the problem in this limit.

c). The transition between the short time and long time behavior occurs for some t_c where the boundary layer thicknesses in a and b are of the same order. What's this t_c?

d). If our electrochemical measurement system requires a time of 1 second to determine a decent measure of the reaction rate, the diffusivity is around 10^{-6} cm^2/s, and the wall shear rate is 100 s^{-1}, what is the minimum length L of the probe? What would be the characteristic boundary layer thickness under these conditions?



$$\frac{\partial c}{\partial t} + \frac{\tau_w}{\mu} y \frac{\partial c}{\partial x} = D\left[\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2}\right]$$

$$\frac{Q}{W} = \int_0^L -D \left.\frac{\partial c}{\partial y}\right|_{y=0} dx$$

Problem 4. (30 points) Pump Curves / Additional Readings / Short Answer:

The first seven questions refer to the pump curve on the last page:

1. It is desired to pump 100 liters/sec from a pond to an elevation of 60 meters. If we neglect all frictional losses (say we use a really fat pipe!) is the pump HH150 recommended for the job?

2. What is the RPM required to do the job?

3. What is the mechanical work done <u>by</u> the pump <u>on</u> the fluid per unit time?

4. What is the efficiency of the pump at the operating conditions?

5. How far up the hill from the level of the pond can we put the pump? (Again, neglect frictional losses) (Note: 1atm ≈ 10.3 m water)

6. Frictional losses always add to the required head. What additional head losses can we tolerate before the pump is unable to achieve the required flow rate?

7. It is proposed to use a 10cm diameter pipe for this system. If we include just the losses due to the initial contraction and acceleration of the fluid, how does the answer to question 5 change?

8. The displacement thickness is defined as:

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy$$

Provide a brief physical interpretation of this quantity.

9. Briefly describe one method for preventing stall on an aircraft wing.

10. Give a physical description of the Reynolds stress (e.g., where does it come from, and how is it defined?).

11. In our class demonstration, when we allowed a rod to settle in a viscous liquid (Re = 0), we observed that it didn't rotate. If inertial effects are important, it does. Why is the behavior different in this case?

12. For a shear stress of 16 dynes/cm^2 in the turbulent flow of water through a pipe, about how rough does the pipe wall have to be before it influences the flow?

13. Why do dimpled golf balls and fuzzy tennis balls have less drag than their smooth counterparts? One sentence, please.

14. Provide two interpretations of $\underset{\sim}{u}$.

15. Provide two interpretations of $\rho\underset{\sim}{u}$

Technical Data

HH150

DIESEL/ELECTRIC AUTO PRIME HIGH HEAD PUMP

SYKES PUMPS

MATERIALS OF CONSTRUCTION

Pump Casing:	S.G. IRON
Suction Cover:	S.G. IRON
Air Separation Tank:	S.G. IRON
Bearing Bracket:	S.G. IRON
Pump Shaft:	431 Stainless Steel
Impeller:	316 Stainless Steel
Wearplates:	316 Stainless Steel
Mechanical Seal:	Silicon / Silicon
N.R.V.:	S.G IRON

DESIGN DETAILS:

Pump Description:	Single Stage, volute type, 3 bladed fully open Centrifugal Pump
Suction Flange:	200 mm
Delivery Flange:	150 mm
Nominal Casing Thickness:	18 mm
Solids Handling Size:	38 mm
Operating Speed:	MIN: 1400 rpm MAX: 2000 rpm
Maximum Head:	98 m
Maximum Capacity:	150 L / sec
Fuel Consumption:	15.5 l/hr

PUMPSET DIMENSIONS

Width:	1250 mm	Height:	1700 mm
Length:	2750 mm	Dry Weight:	1850 kg