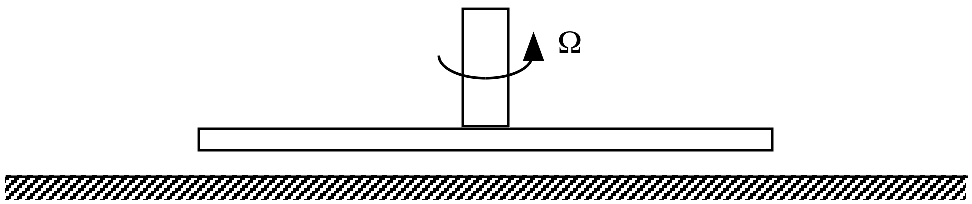
CBE 30355 Transport Phenomena I
Final Exam

December 14, 2015

Closed Books and Notes

Problem 1). (20 points) Lubrication/Scaling: This semester we have examined the flow resulting from a rotating disk. If the disk is far from a surface there is no axial force on it (only torque). If it is very close to a plane, however, the axial force can be very large! Here we examine this in the lubrication limit H/R << 1 where H is the separation and R is the radius, as is depicted below.



a. Calculate the θ velocity in this limit (this is simple, and you've done it before!).

b. Write down the equation governing the radial velocity and pressure distributions in the lubrication limit. (Hint: don't forget to include the one inertial term which actually drives the radial flow!)

c. Write down all the boundary conditions which govern the radial velocity and pressure distributions. (Hint: one is an integral!)

d. Scale pressure and radial velocity, rendering the problem dimensionless. Use this to determine the scaling for the radial velocity, the pressure, and the axial force on the disk.

e. A classmate asserts that you also should also require $H/(\nu/\Omega)^{1/2} << 1$ to be in the lubrication limit. Qualitatively describe what would happen to the velocity profile and axial force on the disk if this were violated. Be brief!

The following equations *may* be helpful (particularly if you circle the dominant terms!):

$$\frac{1}{r}\frac{\partial(r\,u_r)}{\partial r}+\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}+\frac{\partial u_z}{\partial z}=0$$

$$\rho\left(\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta}+u_z\frac{\partial u_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2}+\frac{\partial^2 u_z}{\partial z^2}\right]+\rho g_z$$

$$\rho\left(\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+u_z\frac{\partial u_\theta}{\partial z}+\frac{u_r u_\theta}{r}\right)=-\frac{1}{r}\frac{\partial p}{\partial \theta}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(r\,u_\theta)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2}+\frac{\partial^2 u_\theta}{\partial z^2}+\frac{2}{r}\frac{\partial u_r}{\partial \theta}\right]+\rho g_\theta$$

$$\rho\left(\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}+u_z\frac{\partial u_r}{\partial z}-\frac{u_\theta^2}{r}\right)=-\frac{\partial p}{\partial r}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(r\,u_r)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}+\frac{\partial^2 u_r}{\partial z^2}\right]+\rho g_r$$

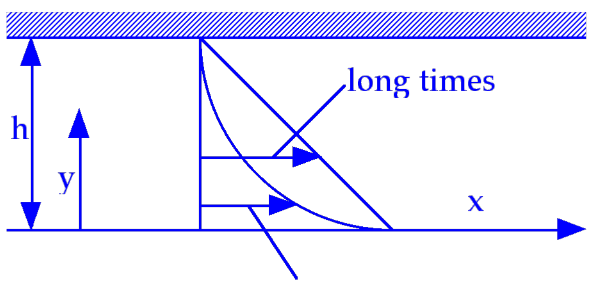
Problem 2. (20 points) Uni-directional Startup Flows. Consider the channel depicted below. For all times $t < 0$ the lower plate and fluid are at rest. At $t = 0$ the lower plate is accelerated such that the shear stress at the wall is a constant τ_w independent of time. The flow may be regarded to be unidirectional in the x-direction.

a. Determine the differential equation and boundary conditions governing this problem.

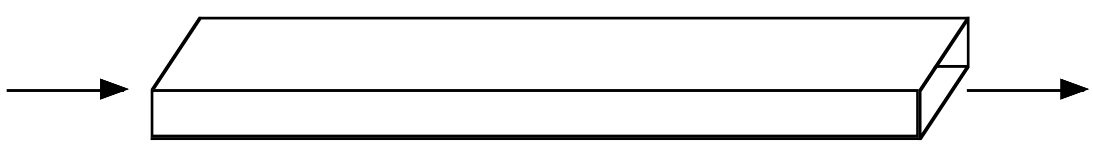
b. At very long times the flow reaches steady-state. Using t_c as this time scale and h as the length scale, how large does t_c have to be for the flow to become steady? (Hint: use scaling analysis and figure out what dimensionless group has to be large or small for the appropriate term to be thrown away). What is the velocity profile in this limit?

c. For very short times a boundary layer develops near the accelerating plate. Using t_c as this time scale, determine the characteristic thickness of this boundary layer (e.g., the new length scale) and the characteristic velocity of the lower plate.

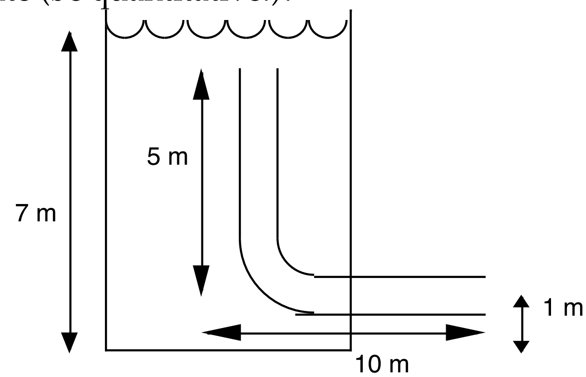
d. Using the technique of coordinate stretching, show that this short-time limit admits a self-similar solution and obtain the similarity rule and variable in canonical form. Determine the velocity of the lower plate as a function of time to within an unknown O(1) multiplicative constant (e.g., the solution of the similarity problem). You don't need to get the transformed ODE to do this!



Problem 3. (10 points) Plane Poiseuille Flow: A problem which is currently being investigated in bioengineering laboratories is the phenomenon of cell adhesion to surfaces in the presence of hydrodynamic stresses. This is very important in the design of biocompatible materials, for example. To study this, a researcher has built a rectangular flow cell which is $50\mu m$ deep, 1mm wide, and 2cm long. The objective is to have a wall shear stress (e.g., stress at the lower wall - the 1mm x 2cm surface - where cell adhesion is being studied) of 10 dyne/cm^2. If the working fluid has the same viscosity as water, what should the flow rate of the pump supplying the fluid be? Express your answer in μl/min. (Hint: While you can derive everything, of course, this problem is much faster if you remember the ratio of centerline to average velocities for plane-Poiseuille flow!)



Problem 4. (10 points) You are designing an overflow drain for a tank as depicted below. It is required that the pipe must handle a flow rate of 20 liters/s. It is proposed to use a 3 inch ID pipe. Will this be adequate (be quantitative!)?



You may find the following expressions useful:

$$h_L = \frac{\langle u \rangle^2}{2g} \sum K + 4 \; f_f \frac{L}{D} \frac{\langle u \rangle^2}{2g}$$

$$f_f = \frac{16}{Re} \; ; \; \text{Re} < 2100 \qquad f_f \approx \frac{0.0791}{Re^{1/4}} \; ; \; 3000 < \text{Re} < 10^5$$

$$\frac{1}{\sqrt{f_f}} = 4.0 \log_{10}\left(\text{Re}\sqrt{f_f}\right) - 0.40 \; ; \; \text{Re} > 3000$$

Fitting	K value
sudden contraction	0.45
sudden expansion	1.0
90° elbow	0.90

Problem 5. (10 points) Jetlev Flyer Nozzle Optimization: Given your familiarity with the Jetlev Flyer geometry, you are asked to determine the optimum jet nozzle diameter. The supply pipe diameter is D_p (e.g., pipe area $A_p = \pi D_p^2/4$) and the total mass to be supported (hose, flyer, water, etc.) is M_T (both are fixed). We seek to determine the optimum nozzle diameter D_N (e.g., total nozzle area $A_N = 2\,\pi D_N^2/4$ as there are two nozzles!) that will yield a flyer height H. The *frictional losses* in the system (pipe friction factor and fitting losses) are approximately given by:

$$h_L = \frac{\frac{1}{2}\rho\left(\frac{Q}{A_P}\right)^2 [3.4]}{\rho g}$$

a. How does the flow rate Q depend on the nozzle diameter?

b. How does the power required depend on the nozzle diameter?

c. How can you use the results of part b to get an equation governing the optimum nozzle diameter for achieving the desired height at minimum power? (Note that it is *possible* to show that the ratio D_N/D_p is a simple function of the dimensionless ratio $\frac{M_T}{\rho H A_p}$ which, for the flyer, reduces to $D_N/D_p \sim 0.4$, but this takes a bit too much time to do on an exam, alas…)

Problem 6. (30 points) Pump Curves / Short Answer:

The first seven questions refer to the pump curve below:

1. It is desired to pump 100 liters/sec from a pond to an elevation of 60 meters. If we neglect all frictional losses (say we use a really fat pipe!) is the pump HH150 recommended for the job?

2. What is the useful mechanical work done by the pump on the fluid per unit time?

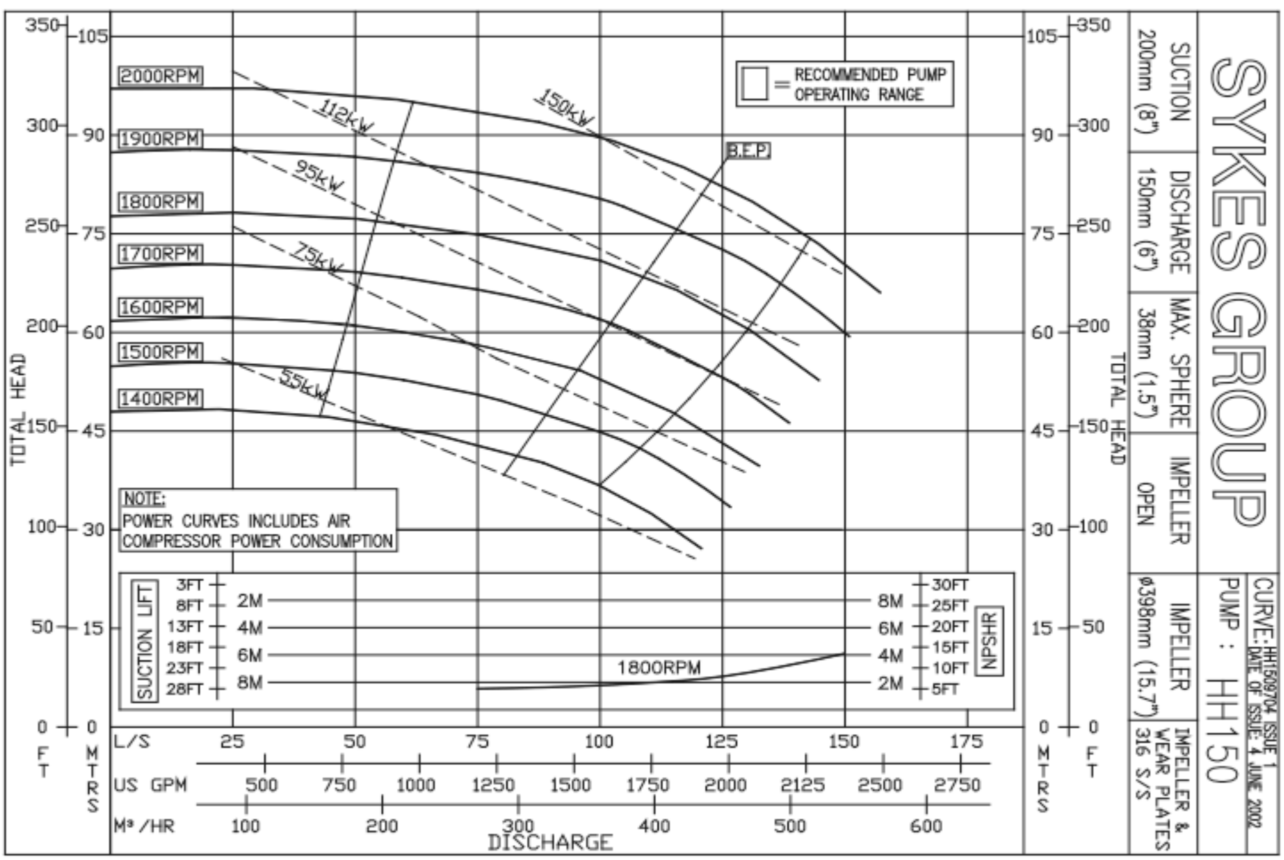
3. What is the efficiency of the pump at the operating conditions?

4. Frictional losses always add to the required head. What additional head losses can we tolerate before the pump is unable to achieve the required flow rate?

5. Your boss proposes to use a 10cm diameter pipe for this system. Quantitatively demonstrate why this is probably a bad idea.

6. About how big should the pipe be instead? Make any approximations you think are reasonable.

7. Suppose the pond is in Denver (at that elevation the pressure is 83% of sea level). Again neglecting all frictional losses, how far up the hill from the level of the pond can we put the pump? (Note: 1atm ≈ 10.3 m water, vapor pressure = 0.3m water)



8. What is the kinetic energy per unit volume of a fluid (in terms of the velocity and material properties)? How does it relate to the pressure at high Re?

9. How is a drag coefficient defined at high Re?

10. Give a physical description of the Reynolds stress (e.g., where does it come from, and how is it defined?).

11. Briefly describe two methods discussed in class for avoiding boundary layer separation on a wing.

12. For a shear stress of 25 dynes/cm^2 in the turbulent flow of water through a pipe, about how rough does the pipe wall have to be before it influences the flow?

13. What is the approximate magnitude of the turbulent Prandtl number (e.g., a Pr based on turbulent flow patterns rather than material properties)?

14. What is the relationship between the Stokes Equations, Stokes' Law and Stokes' Paradox?

15. A sphere of density ρ_m (> fluid density ρ) and radius a is sitting on the wall of a microfluidic channel of half-width b (b>>a). At high enough mean fluid velocities U, inertial lift effects will overcome gravity and cause the sphere to lift off from the wall. **Estimate** this critical velocity. (Hint: in the limit b>>a the lift scales with the shear rate at the wall, not directly with the mean velocity. Use your knowledge of the scaling of inertial forces! Calculation of the lift coefficient was actually my first task as a graduate student many years ago…)