CBE 30355 Transport Phenomena I
Final Exam

December 20, 2019

Closed Books and Notes

Problem 1. (20 points) Scaling of the boundary layer equations. In class we solved for both flow past a flat plate at zero incidence (the Blasius problem) and stagnation flow such as you would get near the leading edge of a cylinder (flow perpendicular to a plate). These are two limits of the more general flow into a wedge which leads to the Falkner-Skan equation. Here we examine this problem.

Consider the flow into a wedge with internal angle of 120° as depicted below. The inner limit of the Euler flow solution has a velocity given by:

$$u^{EF}\Big|_{y/L\to 0} = \lambda x^{1/2}$$

a. Determine the pressure gradient in the boundary layer for this flow. Does this promote or retard the growth of the boundary layer thickness?

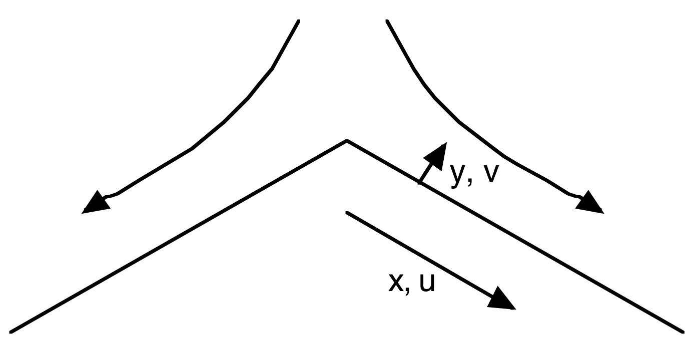
b. Scale the momentum and continuity equations in the boundary layer limit.

c. How does the boundary layer thickness vary with the parameters of the problem (and how does it depend on x, the distance from the leading edge)?

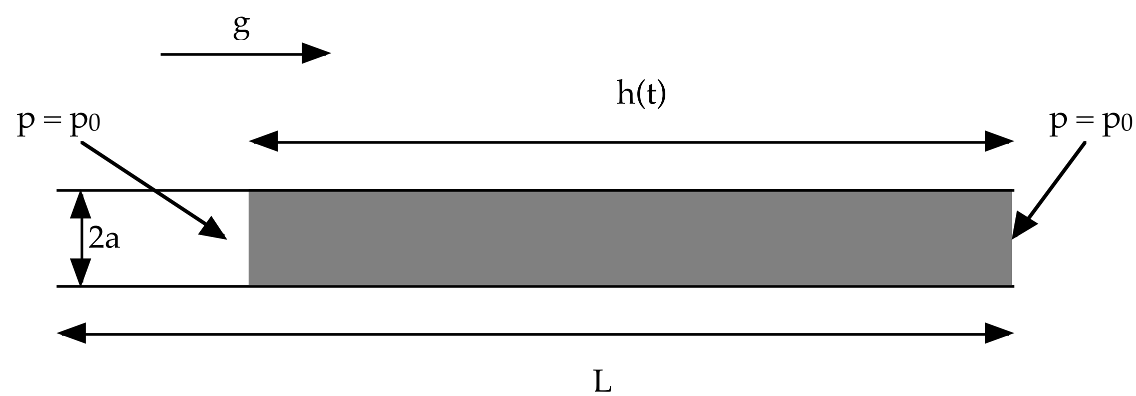
d. How does the shear stress at the surface (y = 0) depend on the parameters of the problem (and how does it depend on x)?

e. How far from the leading edge of the wedge do we have to be before the boundary layer solution becomes valid?

f. Using the continuity equation to define the streamfunction, determine the similarity rule for the streamfunction and the similarity variable (in canonical form!) which would yield the self-similar solution for this problem. Note that I'm not asking you to derive the Falkner-Skan equation (e.g., the transformed ODE) itself, as that gets a little messy…



Problem 2. (20 pts) Scaling/Unidirectional flows: Consider a **vertical** straw of length L and radius a containing a liquid with viscosity μ and density ρ as depicted below (the diagram **is sideways so it fits on the page**!!!). The liquid is initially at rest (we put our finger over the top of the straw). At time t = 0 we remove our finger and allow the liquid to drain out of the straw due to gravity. The length of the straw filled with fluid at any time t is given by h. In this problem we wish to determine the time T_d required to empty the straw - e.g., how long does it take for h to reach zero.



a. A friend argues that T_d is a function of L, a, g, and fluid properties. Use dimensional analysis to determine a dimensionless expression for T_d in terms of these parameters (e.g., construct relevant π groups).

b. Using the unidirectional flow approximation, write down the differential equation governing the fluid velocity (hint: unsteady, in general), keeping only the non-zero terms. Also write down the relation between the velocity and the change in length with time dh/dt of the column of fluid in the straw of length L. Write down all relevant boundary conditions and initial conditions.

c. Scale the equations for HIGH Reynolds numbers, and determine the (unknown) characteristic drainage time t_c in this limit. What boundary/initial condition(s) have to be thrown out in this limit, and what dimensionless group of parameters has to be small?

d. Solve for the drainage time T_d in this limit.

e. Scale the equations for LOW Reynolds numbers, and determine the characteristic drainage time t_c in this limit. You don't need to get the explicit expression for T_d.

The following equations *may* be helpful:

$$\frac{1}{r}\frac{\partial(r\,u_r)}{\partial r}+\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}+\frac{\partial u_z}{\partial z}=0$$

$$\rho\left(\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta}+u_z\frac{\partial u_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2}+\frac{\partial^2 u_z}{\partial z^2}\right]+\rho g_z$$

Problem 3. (10 points) The University is installing a hydroelectric generation system on the St. Joseph River at Seitz Park on the East Race (you may have noticed all the construction down there this fall). The amount of power that can be generated depends on the flow rate, the total head (change in elevation) and turbine efficiency. The average flow rate of the river is 3000cfs (cubic feet per second), of which 600cfs in total is reserved for the waterfall, East and West races, and fish ladder (e.g., other purposes). The change in elevation is approximately 3m.

a. If the turbine efficiency is 80% (a typical value for this size installation), what is the expected power output of the system in MW? (unit conversion: 1 cubic ft = 28.3 liters).

b. The current cost of electricity in Indiana is 0.10 $/kwhr. Based on this, what would be the expected value to the university of the electricity produced in one year?

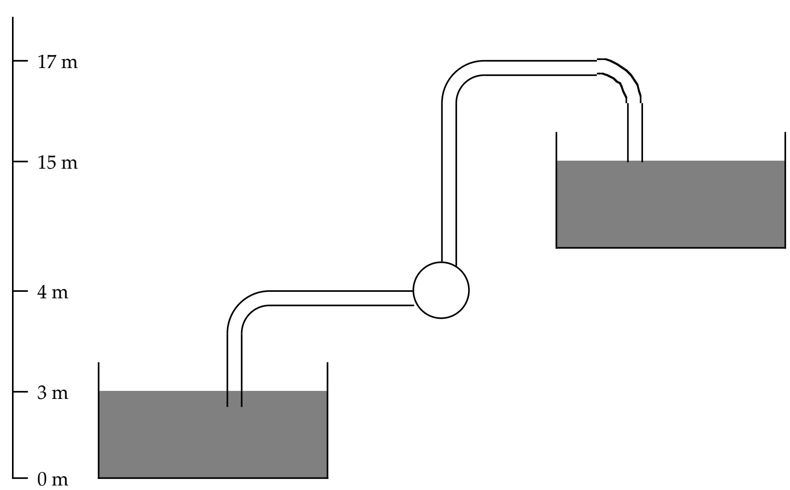
Problem 4. (15 points) It is desired to pump water from a pond into a storage tank using the piping network below (pipe lengths not to scale). The requirements are a flow of 15 liters/s, and the proposed pipe diameter is 7.8 cm ID (e.g., a 3" schedule 40 PVC pipe). The total pipe length is 40 meters.

a. Calculate the total head required of the pump. Correlations for friction factors in pipes and fittings are given on the next page.

b. It is proposed to use the pump CP-80i (pump curve is on the next page). Identify the operating point on the curve, and use this to determine the power requirement and the efficiency of the pump under these conditions.

c. How close to the surface of the pond (vertical elevation) is it necessary locate the pump? (Include 10m length of pipe and other relevant losses)

d. If the cost of electricity is as given above, what is the daily operating cost of the pump which just goes into losses (e.g., the cost you could avoid with a really fat pipe if you were willing to spend more on capital costs)?



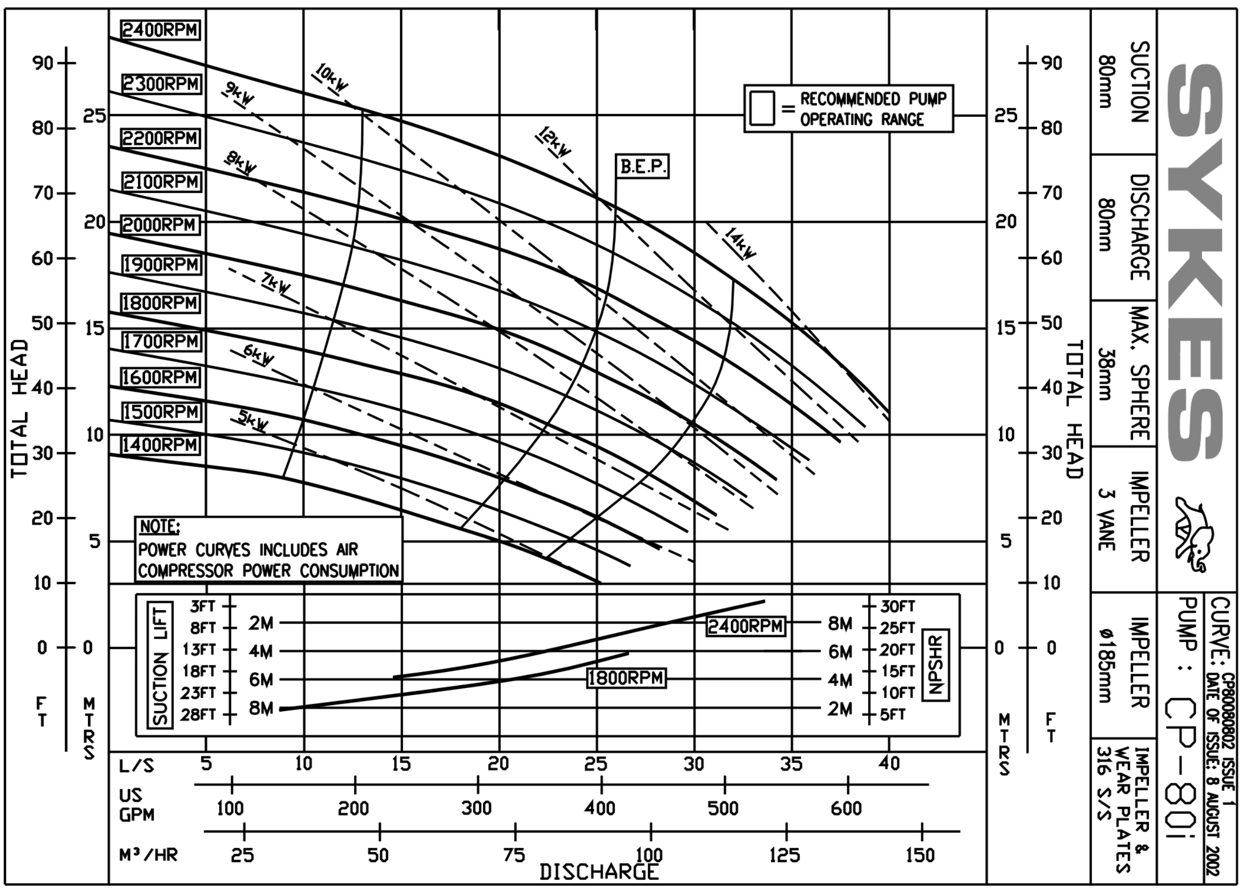
Note: Drawing not to scale (except elevations)!

$$h_L = \frac{\langle u \rangle^2}{2g} \sum K + 4 \quad f_f \frac{L}{D} \frac{\langle u \rangle^2}{2g}$$

$$f_f = \frac{16}{Re} \ ; \ \mathrm{Re} < 2100 \qquad\qquad f_f \approx \frac{0.0791}{Re^{1/4}} \ ; \ 3000 < \mathrm{Re} < 10^5$$

$$\frac{1}{\sqrt{f_f}} = 4.0 \log_{10}\left(\mathrm{Re}\sqrt{f_f}\right) - 0.40 \ ; \ \mathrm{Re} > 3000$$

Fitting	K value
sudden contraction	0.45
sudden expansion	1.0
90° elbow	0.7



Problem 5). (20 points) Short Answer:

1. What is the purpose of the “Trailer Tail” discussed in class and that you often see on trucks on the freeway? (Briefly describe what it is supposed to do!)

2. Using Stokes law, obtain the Stokes sedimentation velocity of a negatively buoyant sphere (e.g., $\Delta\rho > 0$) of radius a settling through a fluid of viscosity μ at low Reynolds numbers.

3. Give a physical description of the Reynolds stress (e.g., where does it come from, and how is it defined?).

4. For a shear stress of 4 dynes/cm^2 in the turbulent flow of water through a pipe, about how rough does the pipe wall have to be before it influences the flow?

5. What do the dimples on a golf ball do?

6. Why is it important to tie a roof down to the foundation if you live in Florida? Be specific!

7. What is the Magnus effect, and where does it come from?

8. What is electroosmosis, and where does it play an important role?

9. What is the “Law of the Wall”? What is the key approximation that leads to it?

10. How does a Jeffrey’s Orbit differ between a rod and a sphere in a simple shear flow?