CBE 30355 Transport Phenomena I
Final Exam

May 15, 2021

Closed Books and Notes

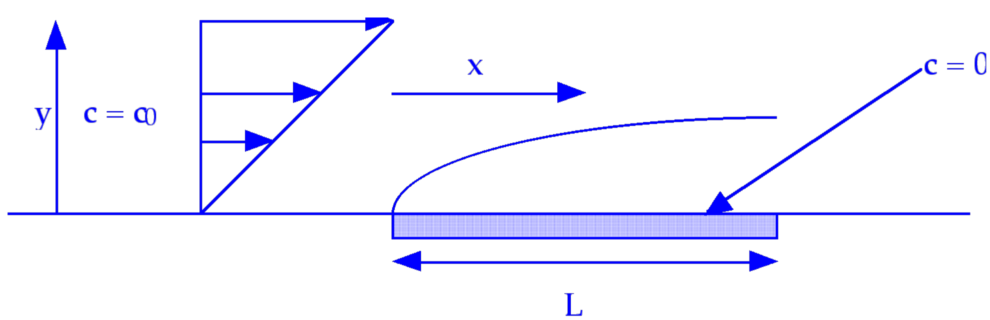
Problem 1. (20 points) Transient Boundary Layer Scaling. You are asked to design an electrochemical probe capable of measuring both concentration and wall shear stress next to a wall. The idea is that fluid flows along the surface of the wall containing some concentration c_0, and then at the surface of the probe we electrochemically reduce the concentration to zero over the whole length L. Initially, you develop a transient boundary layer independent of x, and then at long times you develop a steady, x-dependent boundary layer independent of t. The total reaction rate per unit width (extension into the paper) Q/W can thus be used to determine both c_0 and τ_w by examining these two limits.

a). Scale the equations and boundary conditions at short time scales t_c to determine how the transient boundary layer thickness δ_t and Q/W depend on the parameters of the problem.

b). Scale the equations and boundary conditions at long times to determine how the steady boundary layer thickness δ_s and Q/W depend on the parameters of the problem in this limit.

c). The transition between the short time and long time behavior occurs for some t_c where the boundary layer thicknesses in a and b are of the same order. What's this t_c?

d). If our electrochemical measurement system requires a time of 1 second to determine a decent measure of the reaction rate, the diffusivity is around 10^{-6} cm^2/s, and the wall shear rate is 100 s^{-1}, what is the minimum length L of the probe? What would be the characteristic boundary layer thickness under these conditions?



$$\frac{\partial c}{\partial t} + \frac{\tau_w}{\mu} y \frac{\partial c}{\partial x} = D\left[\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2}\right] \qquad \frac{Q}{W} = \int_0^L -D\left.\frac{\partial c}{\partial y}\right|_{y=0} dx$$

Problem 2. (20 points) Falling (slug) rheometry (Low Re/ lubrication flows): In the class demonstration of falling ball rheometry, it was shown that for a finite sphere diameter/cylinder diameter ratio, the proximity of the walls had a large effect on the sedimentation velocity. This arose from two effects: First, the proximity of the wall increased the shear stress at the surface of the sphere due to the imposition of a no-slip boundary nearby. Second, the cylinder bottom caused there to be a backflow in the gap between the sphere and the cylinder walls due to the downward motion of the sphere. This wall effect is very complicated to derive, however here we look at a vastly simpler problem: what is the sedimentation velocity of a cylindrical slug of length L and radius a centered in a cylinder of radius R, where the gap between the two is $h = R-a \ll a$.

a. If the slug has a density ρ_s and the fluid has a density ρ, what is the net gravitational force on the slug? (hint: don't forget Archimedes' Law…)

b. Now consider the case where the cylinder is open at both ends (say it is completely immersed in the fluid, not exactly practical!). No pressure gradient would be produced by the slug motion in this case. If the slug moves with velocity U_s, calculate the force on the slug due to shear in the gap and, by equating this force with the gravitational force calculated in (a), determine how U_s depends on the parameters of the problem.

c. Now we do the more realistic problem of a closed end cylinder. The downward displacement of the slug leads to an upward displacement of the fluid in the gap and a large pressure difference due to this lubrication flow. While the force due to the shear stress in the gap contributes (and is even greater than that calculated in (b)), by far the dominant contribution for $h \ll a$ is the pressure difference acting on the ends of the slug. Using this insight, calculate the sedimentation velocity of the slug in a closed end cylinder to leading order in h/a.

Problem 3. (20 points) Dimensional analysis and scaling. It is proposed to examine the mixing properties of the H. B. Fuller PB01 mixing tank using strict dynamic similarity. The tank is 1m in radius and 3m high, and the impeller operates at a rotation frequency of 40 rpm. The fluids they are mixing are very viscous, with a viscosity of about 200 poise and a density of 1 g/cm^3. It is proposed to use a 1:9 scale model for the study.

a. The mixing impeller pretty much spans the tank radius (to scrape the goo off the sides). Based on this, what is the approximate magnitude of the Reynolds number and the Froude number in this system? (Hint: don't forget you need to convert rpm to radians/s to get a good estimate…)

b. To achieve strict dynamic similarity the simulant fluid used in the scale model must be changed. What fluid property matters, and what should its new value be?

c. What should be the rotational frequency of the impeller in the scale model?

d. In designing the model system it is necessary to figure out what size motor to buy to drive the shaft. Because we are preserving strict dynamic similarity, we can use the power requirement of the full scale system to determine the motor needed for the model. What is the scale down ratio?

Problem 4. (20 points) Pumped Hydroelectric Energy Storage (PHES) is the most widely used energy storage method in the world. The idea is that when your electricity cost is low (e.g., when the sun is shining on all your solar cells or the wind is blowing on your wind turbines) you use the excess generating capacity to pump water to a high elevation, and when it is more costly (or when the sun isn't shining) you run the system backwards to get your electricity back. These systems can be huge: currently the largest in the world is in Bath, Virginia with a generating capacity of 3GW, enough to power 750,000 homes. Large scale systems are quite cost efficient, with a round trip energy efficiency of 70% to 80% and a relatively low capital cost/GW.

While large systems are economically viable (provided you happen to have a nice mountain with a water source nearby), they have also been proposed (and actually implemented) for individual buildings – but they are not nearly as cost effective. Here we examine some of the numbers associated with a PHES installation in an apartment building in France where one was installed (the Goudemand Residence Complex).

a. The two reservoirs (an open reservoir on the roof and storage tanks in the basement) are separated by 30m in elevation and have an exchange volume of 50m^3. Based on this, what is the total amount of electrical energy that the system can provide (assume that the turbine system converting the potential energy of the water back to electricity is about 75% efficient)? Give your answer in kWhr (unit conversion 1kWhr = $3.6\text{x}10^6$ J).

b. The pumping system is required to pump water from the basement to the roof at a flow rate of 4 liters/s. How many hours would it take to "fully charge" this battery?

c. The pipe system has not been disclosed, however we shall take it to be 50 meters of 2 inch schedule 40 PVC pipe (ID 5.25 cm), plus a safety grate, a sudden expansion, 8 90° elbows, two T's straight through and an open gate valve. Based on this, what is the total head that the pump has to supply?

d. Mark the operating point on the pump curve below (you will have to put your name on the exam paper and turn it in with the bluebook) and determine the required energy input to the pump. What fraction of this energy actually gets stored as potential energy? What is the return trip efficiency of the system (e.g., electricity out / electricity in, assuming the generation side is 75% efficient)?

e. What is the maximum head loss leading to the pump which could be tolerated before violating NPSHR and cavitating the pump?

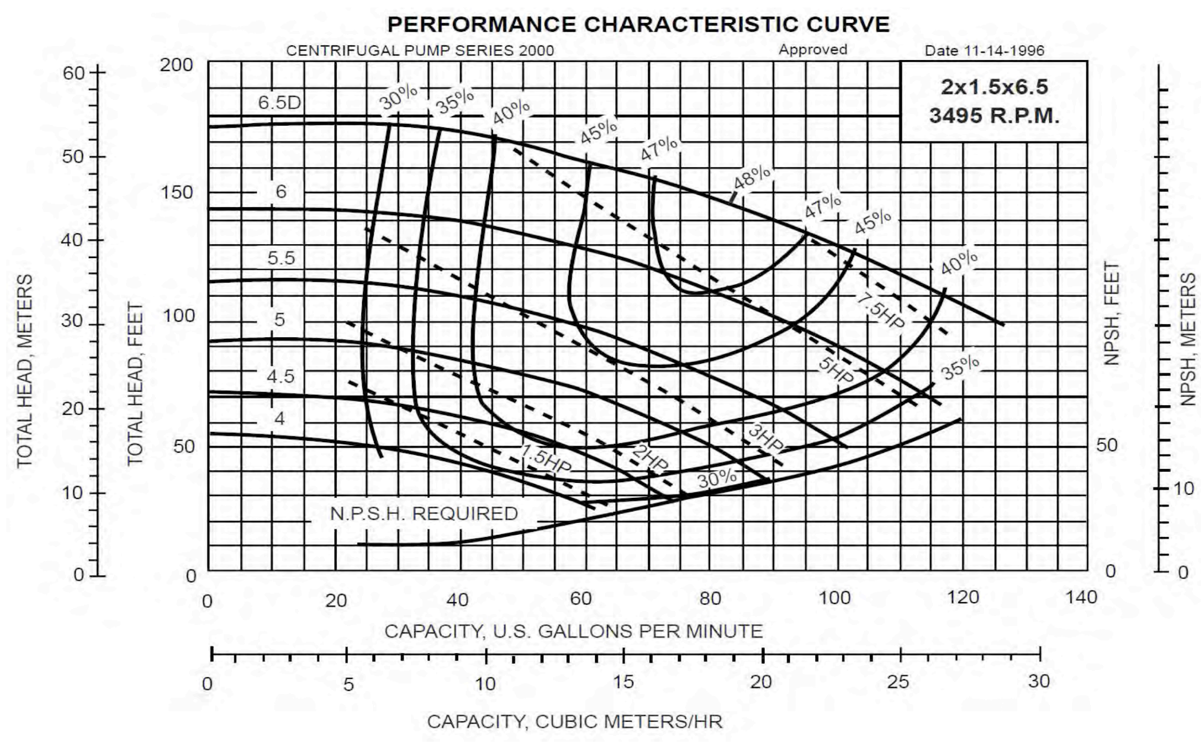
f. This apartment building was really a social experiment in energy utilization rather than trying to be cost effective, however we can look at the economics. To calculate the economics we shall assume as a best case scenario that the pump energy is "free" (it is excess energy from wind turbines and solar cells in this installation) and that the reservoir is fully cycled through the system on average once per day. If the cost of electricity is $0.15/kWhr, what is yearly value of the energy stored by the system? The PHES system was estimated to cost $40k. How many years would it take to recover the capital cost under this idealized (no recurring maintenance or energy costs) scenario? (Hint: get it right, but don't be disturbed if you find it is a really long time…)

$$h_L = \frac{\langle u \rangle^2}{2g} \sum K + 4 f_f \frac{L}{D} \frac{\langle u \rangle^2}{2g}$$

$$f_f = \frac{16}{Re} \ ; \ \mathrm{Re} < 2100 \qquad\qquad f_f \approx \frac{0.0791}{Re^{1/4}} \ ; \ 3000 < \mathrm{Re} < 10^5$$

$$\frac{1}{\sqrt{f_f}} = 4.0 \log_{10}\left(\mathrm{Re}\sqrt{f_f}\right) - 0.40 \ ; \ \mathrm{Re} > 3000$$

Fitting	K value
Safety grate	1.5
sudden expansion	1.0
90° elbow	0.7
T (straight thru)	0.4
Gate valve (open)	0.15



Note: This pump curve gives the power requirement in HP (unit conversion: 1 HP = 0.7457 kW). The curved lines are pump efficiencies – very convenient. The flow rate is in gal/min (unit conversion: 1 liter/s = 15.85 gal/min). And, of course, 1 meter = 3.28 ft.