CBE 30355 TRANSPORT PHENOMENA I TTh 11:00

Second Hour Exam

Closed Books and Notes **November 6, 2018**

Note: Problems are not equally weighted!

Problem 1). (10 points) Dimensional Analysis/Inertial Flows: In class students demonstrated how a can filled with water vapor could be crushed by flipping it over and immersing it in cold water. The idea was that the vapor in the can would rapidly condense, and the water would have to rush into the hole on the end to fill the volume previously occupied by the vapor. The volume of the cans was about 500ml, and the crushing pressure was about $\Delta p = 4\text{x}10^4$ dynes/cm^2. Whether or not a can crushed depended on the size of the hole at the top (bottom when inverted in the water). If a can with a hole area of 5cm^2 crushed while one with area of 10cm^2 didn't crush, *estimate* an upper and lower bound for the time it took for the water vapor in the can to condense when it was immersed in cold water. Don't worry about factors of two…

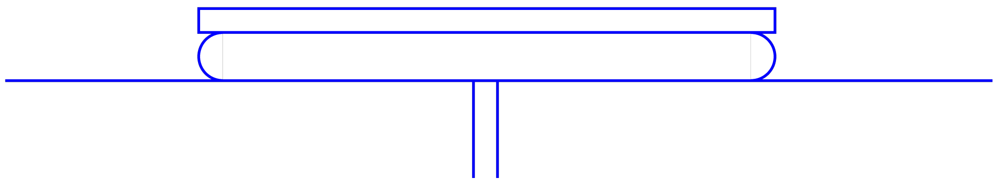
Problem 2). (10 points) Dimensional Analysis: Dip coating with a viscous fluid. Dip coating (where an object is dipped in and then withdrawn from a vat of fluid, leaving a residue of fluid on the surface) is a commonly used coating technique.

a. For the case of strong surface tension Γ (the most commonly encountered case), the thickness of the film δ_f is a function of the vertical velocity U, the viscosity μ, the surface tension Γ (units of Force/Length), and the buoyant force ρg (inertia is small, so it doesn't depend on inertia (density) separately). Using dimensional analysis, determine the number of independent dimensionless groups involved in the problem, and show what group(s) the problem depends on. (Hint: This is fairly simple if you used the dimensional matrix to construct characteristic lengths (groups of parameters with units of length) or reference velocities, and then the dimensionless groups are just ratios of these length or velocity scales.)

b. An interesting paper (Snoeijer, et al., Phys Rev. Let., 2008) demonstrated that under some conditions thicker films can also occur in dip coating. In this case, the thickness no longer depends on Γ. Using this information, repeat the dimensional analysis of part (a) and determine this maximum thickness to within an O(1) constant.

Problem 3). (10 points) A microchannel of length 4cm, width 2mm and depth 100μm is used to study the adhesion of cells to the lower wall. If the required stress at the lower wall is 10 dynes/cm^2 and the working fluid is water, what is the required flow rate? Give your answer in microliters/min (and show your work!).

Problem 4). (20 points) Scaling Theory: Low Reynolds Numbers to High! Consider a disk of radius R located a distance H above a plane as depicted below. Fluid of viscosity μ and density ρ is pumped into the center of this geometry through a small orifice at a rate Q (volume/time), and flows outward radially. It is observed that at low flow rates the disk is blown away from the plane, while at high flow rates it is sucked downwards (as demonstrated in class). Here we analyze this problem in the limit $H/R \ll 1$.



a. Using a mass balance, determine the average radial velocity (averaged over z) and how it depends on r.

b. Via scaling analysis of the equations of motion and a force balance on the plate, determine how the force scales with the parameters of the problem at low flow rates (lubrication limit).

c. Calculate the pressure distribution and force on the plate in this limit. You may leave the force in terms of a dimensionless integral of a known function, although the integral is pretty easy to evaluate to get the final number.

d. Determine via scaling analysis how the force scales with the parameters of the problem at high flow rates (inertial limit). Scaling only here – you would need to know the radius of the feed pipe (not provided) to get the numerical value.

e. If we allow the gap width to change, it will eventually seek a separation where the net force on the plate is zero. Estimate this gap width using scaling analysis.

The following equations may be helpful (note: the vast majority of these terms may be neglected!):

$$\frac{1}{r}\frac{\partial\left(r\, v_r\right)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0$$

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r}$$
$$+ \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\, v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right] + \rho\, g_r$$

Problem 5). (20 points) Short Answer / Multiple Choice:

1. A sphere of radius a has a surface temperature of 1. If the temperature far from the sphere is zero, what is the temperature distribution in the solid surrounding the sphere?

2. What is a yield stress? Name one material which would exhibit it.

3. Which (if any) of the following ***must*** be continuous at a fluid-fluid interface?
   A. Shear rate
   B. Heat flux
   C. Mass flux
   D. Velocity

4. A tube of radius R has a surface roughness of size εR. At high Re the roughness is found to have a large effect on the pressure drop, while at low Re it is only of O(ε). ***Briefly*** explain why the effect is so small at low Re.

5. In ship design, the theoretical maximum velocity of an important class of single hull designs (heavy, deep keel boats) occurs when the Froude number (based on the length of the hull at the waterline) is about 0.16. What is this maximum velocity for a 90m long ship of this type?

Briefly identify the following equations **and a problem where it would play a role**:

6. $D_0 = \dfrac{kT}{6\pi\mu a}$

7. $U_s = \dfrac{2}{9}\dfrac{\Delta\rho g a^2}{\mu}$

8. $\nabla^4 \psi = 0$

9. $Q = \dfrac{-\pi}{8}\dfrac{\Delta P}{L}\dfrac{R^4}{\mu}$

10. $p + \frac{1}{2}\rho u^2 + \rho g h = Cst$