CBE 30355 TRANSPORT PHENOMENA I

First Hour Exam
10/7/14

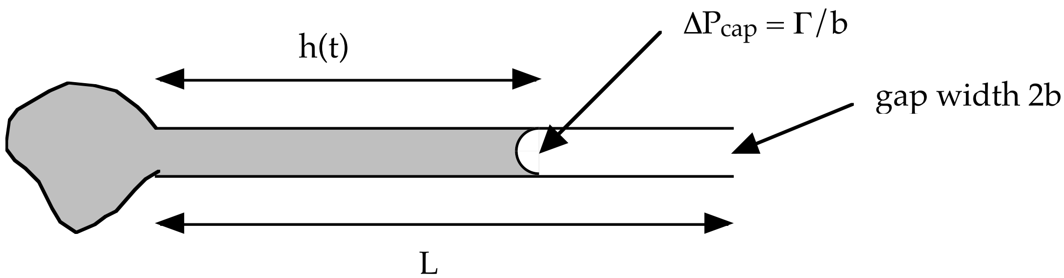
This test is closed books and closed notes

Problem 1. (20 pts) Conservation of Momentum: In a fun WSJ article on water toys for the super-rich, I came across the Jetlev Flyer. It consists of a small boat/pumping unit, a 10m long 4" ID feed tube, and a jet backpack which redirects the water downwards through two 1.5" ID nozzles. According to their advertisements, you can fly for hours (before the modified jetski boat pump runs out of gas), and travel up to 25mph towing the pump boat behind you. Although more than a bit pricey (it costs around the same as a new car!) you can actually get a 15 minute ride on one for less than $200 at a few beaches in Australia. Here we analyze the requirements for the pumping system.

Flyer weight: 100 kg
Jetpack weight: 15 kg
Feed pipe weight: 0.5 kg/m

Using this information, and neglecting all frictional losses (e.g., assume that Bernoulli's equation applies), determine the required flow rate, and the pressure at the **base** of the feed pipe (e.g., the pressure the pump would have to supply) to support our flyer at the maximum altitude of 10m. Don't forget the weight of the water in the feed pipe!

Problem 2. (20 pts) Earlier this semester we briefly looked at capillary pressure effects, whose magnitudes are characterized by the surface tension Γ. One way fluid (e.g., blood) is drawn into a microfluidic channel for analysis is using capillary wetting, the same effect that causes water to wick into a paper towel. In this problem we examine the capillary wetting of fluid in a narrow 2-D channel. Consider the system depicted below. Fluid enters the thin gap of width 2b. If the fluid wets the surfaces of the channel (basically, if a drop of the fluid would spread out over the material composing the channel surfaces), then the curvature of the interface will cause a capillary pressure drop between the fluid and the air of $\Delta p_{cap} = \Gamma/b$. Because the pressure in the fluid behind the meniscus is now *less* than that in the reservoir, it sucks fluid into the gap from the reservoir. After some time T_f the fluid fills the channel (and thus would reach the analysis section). It is this time we wish to determine.



a. For very thin channels inertial (including the term arising from accumulation of momentum!), gravity, and end effects will be negligible and we will get unidirectional flow. If, at some instant in time, the fluid has filled the channel to a length h, calculate the velocity distribution and flow rate.

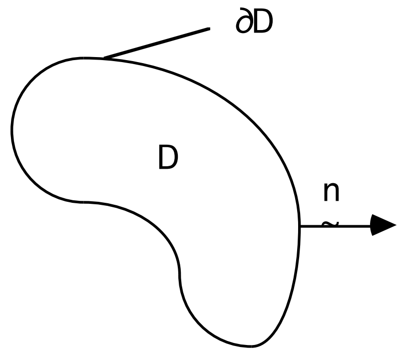
b. Now for the transient problem. Using the result of part a, develop an equation for how h evolves in time (initially h=0, e.g., the channel is empty). Use this to determine the fill time T_f where h = L.

Problem 3). (10 pts) One of the key aspects of our research over the years has been developing equations governing particle concentration distributions in suspensions of particles in fluids. Here we look at the evolution of the particle volume fraction (the fraction of the local volume occupied by the particles) given by the variable ϕ. The particle flux may be modeled as the sum of two terms: the convective flux vector $u_i\phi$ due to the local fluid velocity vector u, and an additional diffusive flux specified by a flux vector N_i. It's closing the problem by relating N_i back to the velocity and concentration profile which is the tricky part of all the research, but we won't worry about that here. Starting from the arbitrary stationary control volume depicted below, derive the conservation equation for the particle volume fraction.

a. Write down the conservation relationship in words. Don't forget accumulation!

b. Write down the conservation relationship in integral form.

c. Complete the problem by deriving the differential form of the conservation law.



Problem 4. (20 pts) Short Answer / Index notation / Additional Readings

1. Which way does Crooke's Radiometer rotate, and why?

2. What is the ratio of the centerline velocity to the average velocity for a) laminar flow in a tube, and b) laminar flow in a channel?

3. What is the continuum hypothesis, and where does it break down?

4. Write down the isotropic part of the total stress tensor in index notation.

5. Write down the continuity equation for a c**ompressible** fluid using index notation.

6. What is a pyroclastic flow, and why is it dangerous?

7. What is the representation of the deviatoric stress τ_{ij} in terms of the velocity for an incompressible Newtonian fluid? (Index notation, please!)

8. Provide two physical interpretations for $\rho \underset{\sim}{u}$.

9. What is the mathematical representation of the coriolis force, and what direction does it act in?

10. What are the mathematical representations of the Reynolds number and the Prandtl number? The product RePr often plays an important role in heat transfer – what is its physical interpretation?