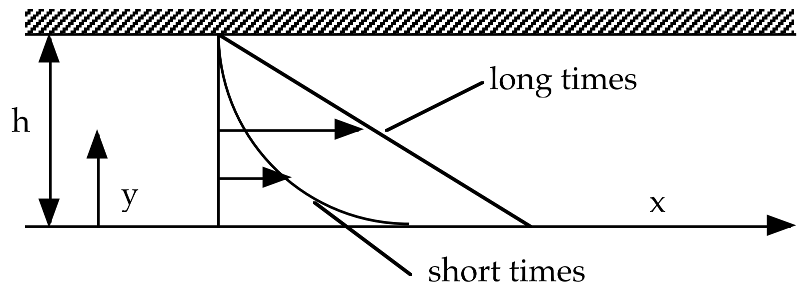
CBE 30355 Transport Phenomena I
Final Exam

November 16, 2020

Closed Books and Notes

Problem 1. (20 points) Unsteady unidirectional flow. Consider the unidirectional time-dependent start-up flow depicted below. Initially the fluid is at rest between two plates separated by a distance h. At time t = 0 the lower wall has a stress F/A applied in the x-direction, causing it to move. The upper wall is fixed (no motion). In this problem we use scaling to examine all the interesting parts of the problem.



a). The starting point: Write down the governing differential equation and boundary conditions valid for all times and associated boundary conditions, crossing out all the terms which are zero. Do not include gravity. Don't forget how F/A yields the boundary condition at the lower wall!

b). Long times: Render the equations dimensionless, using some undetermined timescale t_c for time. Show that for very large t_c (e.g., if we wait a long time), inertia no longer matters, and solve for the asymptotic velocity distribution (which should be very familiar). About how long do we have to wait for this solution to become valid?

c). Short times: For very short times the length scale should no longer be the gap width, but instead should be some boundary layer thickness δ. By rescaling the equations in this limit, how do δ and the velocity scale with t_c?

d). Short times: Using either the results of part c, or via simple affine stretching, show that the short time boundary layer problem admits a similarity solution. Give the similarity rule and the similarity variable in canonical form, and determine the time dependent velocity of the lower wall to within some unknown multiplicative constant (e.g., the solution of the transformed differential equation that you don't have time to get).

e). All times: Using the results of (a) and (d), on the same plot sketch up the velocity of the lower wall as a function of time (rendered appropriately dimensionless) in the two asymptotic limits. Show the domain of validity of the two solutions.

Problem 2. (20 points) Dimensional Analysis/Stokes flow. Last summer I was asked to evaluate whether a face shield would serve as an adequate substitute for a face mask in protection against the emission or inhalation of droplets. Unfortunately, the answer was no. Because a shield is solid, the only way it can filter out droplets emitted during speech is via inertial impaction: because of their mass they do not exactly follow the streamlines of the air deflected by the shield, and instead (if they are big enough) impact on the surface. Here we examine this phenomenon by looking at a one-dimensional analog.

Consider a spherical droplet of radius a which initially moves with the velocity U_0 of the fluid (the exhaled air). At time $t = 0$ you stop the flow (corresponding to deflection by the face shield in the real problem), but due to its mass the droplet keeps going for a while. Inertial impaction occurs if the final displacement relative to the fluid is greater than the distance to the face shield.

a. The displacement of the droplet Δx depends on the density of the drop ρ, the density of the fluid (air) ρ_a, the viscosity of the air μ, the radius of the drop a, and the initial velocity U_0. Using dimensional analysis, determine the dimensionless groups the displacement depends on.

b. The result in part a isn't terribly useful, as there are too many groups! If the droplets are really small, however, (and the ones we are most worried about are really really small!) their motion is governed by Stokes flow (low Re). In this case, the density of the air is negligible and the displacement Δx is proportional to U_0 (e.g., due to the linearity of the governing flow equations). Use this to strengthen your dimensional analysis down to a single group.

c. This still isn't good enough, as we need to actually solve the problem to get the "O(1) constant". Using Newton's Second Law (e.g., force = mass * acceleration!) and Stokes' Law (hint: remember 6π…) for the drag on a sphere, set up and solve for the time-dependent velocity and displacement of a droplet.

d. If the initial velocity is 100 cm/s, the viscosity of air is 1.81×10^{-4} g/cm s, and the distance to the face shield is 5 cm, estimate the diameter of the droplets which could be captured by inertial impaction. The problem is that the droplets emitted in speech are smaller than this, and thus they get away…

Problem 3. (10 pts) Scaling/High Re flows: Hydroplaning of a car occurs when it hits a puddle when moving too fast, and the tires lose contact with the road. Interestingly, the velocity at which this occurs (at least for bald tires) depends primarily on the tire pressure – the weight of the car doesn't matter.

a. Briefly explain why this would occur, and develop an expression for the critical velocity in terms of the tire pressure.

b. If the tire pressure is 36 psig (e.g., 250 kPa), estimate the velocity at which a car with bald tires will hydroplane.

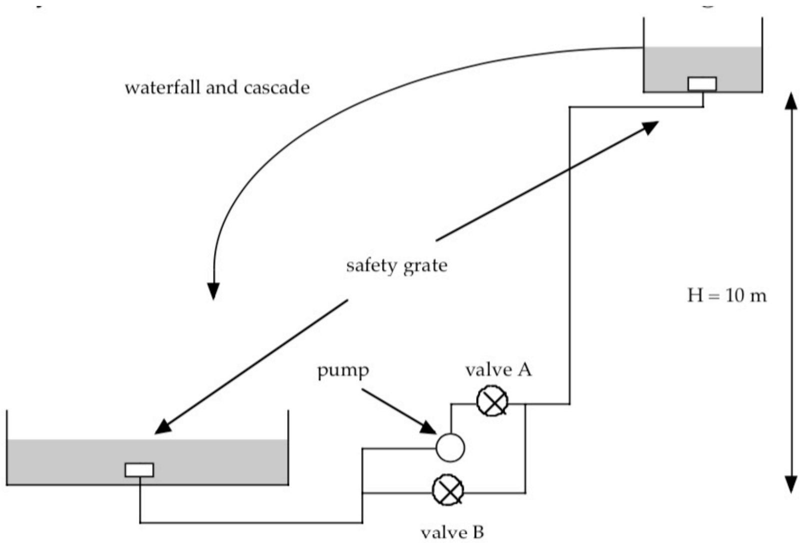
Problem 4. (10 points) The University is installing a hydroelectric generation system on the St. Joseph River at Seitz Park on the East Race (you may have noticed all the construction down there this year). The amount of power that can be generated depends on the flow rate, the total head (change in elevation) and turbine efficiency. The average flow rate of the river is 3000cfs (cubic feet per second), of which 600cfs in total is reserved for the waterfall, East and West races, and fish ladder (e.g., other purposes, so subtract it from the 3000cfs!). The change in elevation is approximately 3m.

a. If the turbine efficiency is 80% (a typical value for this size installation), what is the expected power output of the system in MW? (unit conversion: 1 cubic ft = 28.3 liters).

b. The current cost of electricity in Indiana is 0.12 $/kWhr. Based on this, what would be the expected value to the university of the electricity produced in one year?

Problem 5. (15 points) High Re Flow/Pump Curves: A very attractive part of many public gardens is the water feature (it's my favorite part, anyway). Because most of these gardens aren't blessed with a naturally occurring source of flowing water, the features are usually driven by a pump. Here we size such a system.

Consider the garden feature depicted below (the picture is from the Royal Botanical Garden near Toronto). Water is pumped from a lower reservoir into an upper reservoir which then cascades in a waterfall and a creek back into the lower reservoir. The piping network consists of 50m of 3" ID pipe, and begins and ends with a couple of safety grates (K factor of 1.5 each). The valves are gate valves which can be set either for pump operation (valve A open, valve B closed), or for draining the upper reservoir (valve A closed, valve B open) but we're not looking at drainage this time. Note that a centrifugal pump usually redirects the flow, so you don't have an extra elbow for it in the diagram below.



a. It is desired to run the waterfall at 15 liters/s. What is the energy requirement for the pump CP-80i to provide this flow? Operating costs are mostly energy, which costs \$0.12/kwhr. What is the daily cost (12 hours on average) of operating the waterfall?

b. How far above the lower pond can we site the pump before "bad things happen"? Assume you've got about 10m of pipe leading to the pump.

c. It is proposed to reduce operating costs by using 4" ID pipe rather than 3" ID pipe. The larger pipe currently costs \$2/ft more. Remember how velocity and losses scale with pipe diameter! Using this, estimate (e.g., use the same friction factor, not a terrible approximation) how long reduced operating costs would take to pay for the larger diameter pipe.

$$h_L = \frac{\langle u \rangle^2}{2g} \sum K + 4 f_f \frac{L}{D} \frac{\langle u \rangle^2}{2g}$$

$$f_f = \frac{16}{Re} \;;\; \mathrm{Re} < 2100 \qquad\qquad f_f \approx \frac{0.0791}{Re^{1/4}} \;;\; 3000 < \mathrm{Re} < 10^5$$

$$\frac{1}{\sqrt{f_f}} = 4.0 \log_{10}\left(\mathrm{Re}\sqrt{f_f}\right) - 0.40 \;;\; \mathrm{Re} > 3000$$

Fitting	K value
Safety grate	1.5
Gate valve (open)	0.15
90° elbow	0.7
T (straight through)	0.4
T (through side)	1.5