CBE 30355 Transport Phenomena I
Final Exam

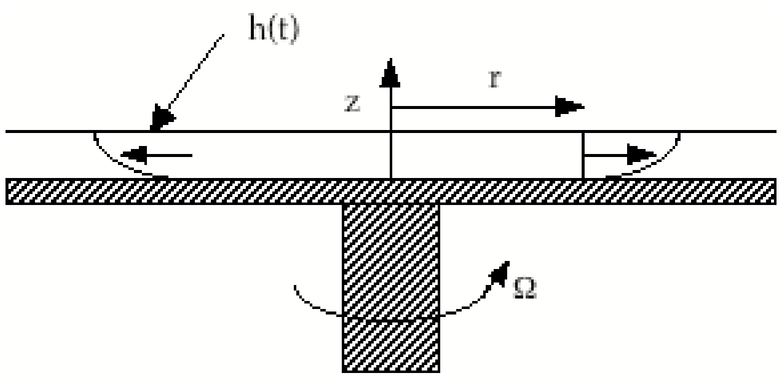
December 12, 2022

Closed Books and Notes

Problem 1. (20 points) Scaling/Lubrication: A common way to produce a thin film on a surface is to spin coat it - take a disk of radius R, deposit some liquid of kinematic viscosity ν on the surface, and then rotate it with some angular velocity Ω. Centrifugal forces cause the fluid layer to thin out over time, leaving a final thickness $h_f << R$ at the end of the process. Here we analyze this procedure.

a). For *thin* fluid layers the rotational velocity is just $u_\theta = \Omega r$ over the entire film (e.g., it moves with the rotational velocity of the disk unlike what occurs in the boundary layer problem you looked at earlier this term). Using this, determine the differential equations and boundary conditions which govern the radial and vertical velocity distribution in the film, and the equation for the change in film thickness over time. (Hint: How does the change in thickness with time relate to u_z at $z = h(t)$?) It is appropriate to use h_f, the final thickness, as the vertical length scale.

b). Using scaling analysis, determine how long the spin coating process takes as a function of the parameters of the problem to within the usual unknown O(1) constant. If you work it out completely you would find the constant to be ¾, however it does get a little messy…



You may find the following equations helpful:

$$\frac{1}{r}\frac{\partial (r\, v_r)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0$$

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r}$$

$$+\,\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\, v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right] + \rho\, g_r$$

Problem 2 (10 pts). Hydrostatics/Archimedes Law: You are given the task of determining the gold content of a "red gold" chain (actually a gold/copper alloy) that a friend has brought back from a trip to the Middle East. The chain weighs 180 g, however when suspended on a fine thread in a beaker of isopropanol (density 0.785 g/cm^3) it weighs out at only 12.95 g. If the density of gold is 19.32 g/cm^3 and the density of copper is 8.96 g/cm^3, what karat is the chain (24 karat = pure gold)?

Problem 3 (10 points). Dimensional analysis/Buckingham Π theorem: As demonstrated in class, when a drop of volume V, density ρ_d and viscosity μ_d is released into a fluid of viscosity μ_f and density $\rho_f < \rho_d$ it will expand into a torus that eventually breaks up after falling a height H. Here we use dimensional analysis to examine this problem.

a. Using the Buckingham Π theorem, determine the number of dimensionless groups that the problem depends on and construct an independent set.

b. Recognizing that ρ_f and ρ_d are very close, strengthen your result by only including ρ_d in the buoyancy term (e.g., $\Delta\rho g$ rather than ρ_d and g separately).

c. Empirically it is observed that the break up height H is nearly independent of the drop volume. Under these circumstances, for fixed viscosity ratios μ_d/μ_f how should H vary with the fluid viscosity μ_f?

Problem 4 (20 points). Scaling/Transient boundary layers: Consider a solid slab of thickness h. The slab is initially at a temperature of zero, and the surface at y = h is kept at that temperature at all times. For all t > 0, however, the slab is heated at y = 0 with a constant heat flux q_0, warming things up.

$$\rho \hat{C}_p \frac{\partial T}{\partial t} = k\nabla^2 T \;; \qquad T\big|_{t=0} = T\big|_{y=h} = 0; \qquad -k\frac{\partial T}{\partial y}\bigg|_{y=0} = q_0$$

a. Write down the equations and boundary conditions governing this problem and render them dimensionless. What is the appropriate scaling for time, and what is its significance? What is the scaling for the temperature?

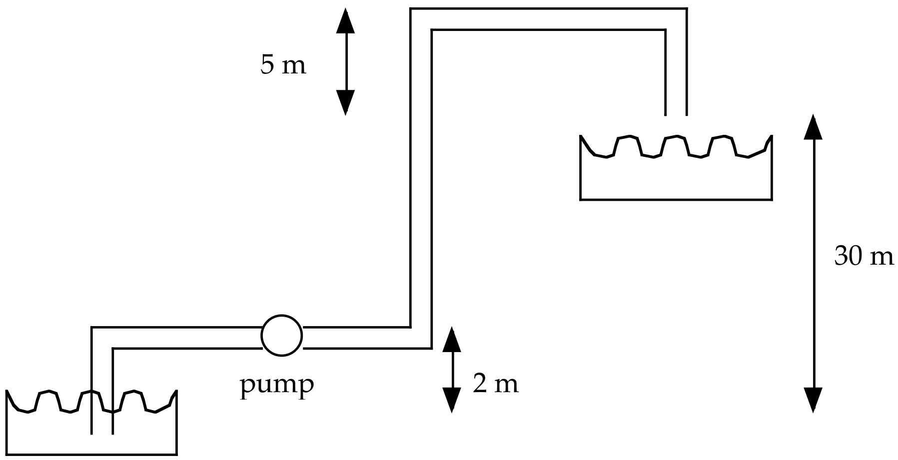
b. Solve for the temperature distribution at large times and get the dimensionless temperature of the lower (heated) surface at y = 0.

c. Now for the fun part: rescale everything for short times (e.g., the transient boundary layer limit). Show that the problem admits a self-similar solution (you don't need to solve it, or even get the transformed ODE, although this one's pretty easy) determining the similarity rule and variable in canonical form.

d. Determine the transient temperature of the lower surface as a function of time to within an unknown O(1) constant (e.g., the solution to your ODE), and plot up how the dimensionless temperature varies with time, including the asymptote from part b.

Remember that heat conduction in solids is mathematically identical to unidirectional fluid flow!

Problem 5). (20 points) Pumps & Pipes! Consider the piping system depicted below. You are pumping water from one reservoir to another 30 m up the hill at a rate of 16 liters/s through 7.5 cm ID pipe. There is 10 m of pipe leading to the pump and another 90 m leading to the second reservoir, with a total of four 90° elbows in the system. The pump you are given to use is the HH80, whose pump curve is on the next page.



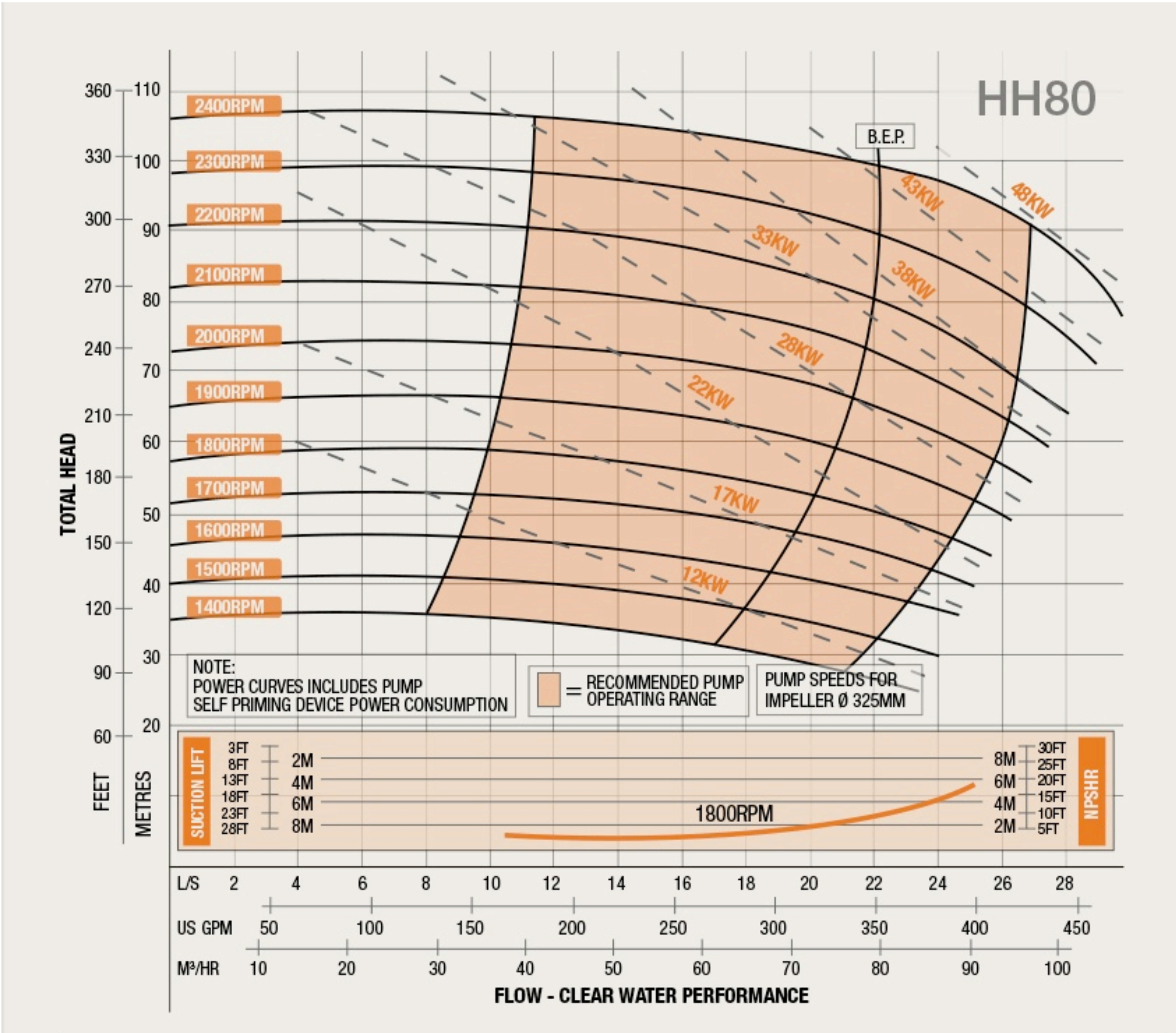
a. What is the total head the pump is required to supply (in meters of H_2O)?

b. What fraction of the energy cost of operating the pump goes into the change in elevation?

c. Will this design violate NPSHR? Be quantitative!

d. A contractor comes by and proposes to replace the 7.5cm pipe with 10cm pipe. He wants to charge $10k for the fix. You are tasked with figuring out if this is a good idea or not. If the cost of electricity is $0.15/kWhr, how long would it take for the reduction in operating costs to pay back the contractor's fee?

e. Based on your calculations, would it make sense to install an even bigger pipe (say 15cm)? A qualitative answer here – don't redo the calculations!



$$h_L = \frac{\langle u \rangle^2}{2g} \sum K + 4 \; f_f \frac{L}{D} \frac{\langle u \rangle^2}{2g}$$

$$f_f = \frac{16}{Re} \;;\; \text{Re} < 2100 \qquad\qquad f_f \approx \frac{0.0791}{Re^{1/4}} \;;\; 3000 < \text{Re} < 10^5$$

$$\frac{1}{\sqrt{f_f}} = 4.0 \log_{10}\left(\text{Re}\sqrt{f_f}\right) - 0.40 \;;\; \text{Re} > 3000$$

Fitting	K value
sudden contraction	0.45
sudden expansion	1.0
90° elbow	0.70