CBE 30355 TRANSPORT PHENOMENA I

Mid-Term Exam
10/12/23

This test is closed books and closed notes

Problem 1 (20 points). Hydrostatics/Inertial Scaling: Submarine implosions are very rare, however as I'm sure you are aware the Titan submersible was destroyed last summer exploring the wreckage of the Titanic. In this problem we are going to examine some aspects of this tragedy. The submersible was a stubby cylinder (that was a big part of the problem!) with an equivalent spherical radius R_0 of 1.1m. It imploded near the Titanic which is at a depth of 3800m. The density of seawater is 1.028 g/cm^3.

a. What was the pressure on the hull in atmospheres at the depth of the Titanic (about where it imploded)? Recall 1atm = 101kPa and the acceleration due to gravity is 9.8m/s^2.

b. What was the energy released by the implosion? (Hint: remember Archimedes law and how energy is related to force and distance!). Give your answer in kg of high explosives: 1kg HE = 4.2x10^{6}J.

c. We are interested in estimating the time necessary for the implosion to occur once the hull is breached. We can model this process by the implosion of a spherical "bubble" with initial radius $R = R_0$. The fluid velocity at r = R(t) (e.g., the surface of the imploding bubble) has some value $u_r|_{r=R} = U(t)$. Using the continuity equation, determine the radial velocity for all r > R in terms of the (unknown) U(t).

d. The change in radius with time dR/dt is just U(t). Initially the pressure inside the bubble is one atmosphere (negligible), while the pressure at large r is the value you got in part (a) from hydrostatics. Using this, scale the radial component of the Navier-Stokes equations in spherical coordinates to get the characteristic implosion time. Interestingly, if you actually solve the problem (including the adiabatic compression of the bubble affecting the pressure inside as well – it oscillates if it is perfectly symmetric and you ignore all losses) you get a dimensionless implosion time of 0.917 – so the value you get from scaling alone matches the complete solution to within 10%!

The following equations are helpful – just remember that the vast majority of the terms are zero!

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(\rho r^2 u_r\right) + \frac{1}{r\sin(\theta)}\frac{\partial}{\partial \theta}\left(\rho u_\theta \sin(\theta)\right) + \frac{1}{r\sin(\theta)}\frac{\partial}{\partial \phi}\left(\rho u_\phi\right) = 0$$

$$\rho\left(\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta} + \frac{u_\phi}{r\sin(\theta)}\frac{\partial u_r}{\partial \phi} - \frac{{u_\theta}^2 + {u_\phi}^2}{r}\right) = -\frac{\partial p}{\partial r}$$
$$+\mu\left[\frac{1}{r^2}\frac{\partial^2}{\partial r^2}\left(r^2 u_r\right) + \frac{1}{r^2\sin(\theta)}\frac{\partial}{\partial \theta}\left(\sin(\theta)\frac{\partial u_r}{\partial \theta}\right) + \frac{1}{r^2\sin^2(\theta)}\frac{\partial^2 u_r}{\partial \phi^2}\right] + \rho g_r$$

Problem 2). (10 points) Bernoulli's Equation/Integral Momentum Balances: You've all seen it: a laminar stream of water emerges from a water faucet and, as it falls downwards, gets thinner and thinner. Here we analyze this problem.

a. If the initial velocity of the water emerging from the faucet is U_0 and stream diameter is D_0, what is the diameter of the stream when it reaches a cup a distance H below the faucet?

b. What it the force exerted by the momentum of the stream on the (overflowing) cup? Make any simplifications you deem necessary, but justify them!

Problem 3). (10 points) Dimensional Analysis: Mixing in a large vessel is usually characterized by the energy input per unit volume and time (e.g., the power going to the impeller divided by the tank volume). To fully simulate the behavior on a computer you have to solve the flow equations over lengths ranging from the tank size down to something called the Kolmogorov scale η, the length scale at which turbulent fluctuations die away.

a. If the Kolmogorov scale depends on this power input P/V and the density and viscosity of the fluid, how does it scale with the parameters of the problem?

b. You are trying to model a mixing vessel filled with glycerin ($\rho = 1.26 g/cm^3$, $\nu = 12 cm^2/s$). The tank has a volume of $1m^3$ and the impeller input has a power of 1kW. What is the approximate numerical value of the Kolmogorov scale? (note: we're not worried about the O(1) constant here, although the accepted value is 0.95…) Approximately how many grid points would be required for a 3-D numerical simulation?

Problem 4). (10 points) Plane Poiseuille Flow: A problem which is currently being investigated in bioengineering laboratories is the phenomenon of cell adhesion to surfaces in the presence of hydrodynamic stresses. This is very important in the design of biocompatible materials, for example. To study this, a researcher has built a rectangular flow cell which is 40μm deep, 1mm wide, and 2cm long. The walls aren't moving, so the flow is being driven by a syringe pump. The objective is to have a wall shear stress (e.g., stress at the lower wall - the 1mm x 2cm surface - where cell adhesion is being studied) of 10 dyne/cm^2. Due to the ratio of length scales, you can assume unidirectional plane-Poiseuille flow. If the working fluid has the same viscosity as water (~1 cP), what should be the flow rate of the syringe pump supplying the fluid?