CBE 30355 TRANSPORT PHENOMENA I

Mid-Term Exam
4/1/21

This test is closed books and closed notes

Problem 1 (20 points). Flow resistance in membranes: unidirectional flows. A microfiltration membrane of thickness h is often modeled as an array of cylindrical pores of radius a. The membrane is further characterized by a porosity (fraction open area) of ε. We are interested in the pressure drop Δp required for a flow rate Q/A (flow rate per unit area, encompassing both the pores and solid).

a. For this model of membrane flow, determine the relationship between Δp and Q/A.

b. For a pore radius of 2μm, membrane thickness of 1mm, porosity ε of 0.5 and the properties of water, determine the pressure drop necessary to achieve a flow rate of 60 ml/(min cm^2). Express your result in atmospheres (hint: 1 dyne/cm^2 is 0.99x10^{-6} atm, and 1μm is 10^{-4}cm).

c. Your calculation in b assumed unidirectional, laminar flow. Quantitatively comment on whether this is appropriate for cylindrical pores of this size at this flow rate.

d. For many applications a pore radius of 2μm is way too large (bacteria would go right through, for example). Thus, we usually use *asymmetric membranes* which have two layers: a support layer with larger holes (such as the parameters in b) and a thinner surface layer with much smaller holes. If we require this surface layer to have pore radii of 0.25μm, how thin would it have to be for the pressure drop to no more than double (e.g., for the thin layer to have the same pressure drop as the thicker layer with the larger 2μm radii pores) for the same flow rate. Give your answer in microns.

The following equations *may* be helpful:

$$\frac{1}{r}\frac{\partial(r u_r)}{\partial r}+\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}+\frac{\partial u_z}{\partial z}=0$$

$$\rho\left(\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta}+u_z\frac{\partial u_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2}+\frac{\partial^2 u_z}{\partial z^2}\right]+\rho g_z$$

Problem 2 (20 points). Dimensional analysis and scaling. It is proposed to examine the mixing properties of the H. B. Fuller PB01 mixing tank using strict dynamic similarity. The tank is 1m in radius and 3m high, and the impeller operates at a rotation speed of 40 rpm. The fluids they are mixing are very viscous, with a viscosity of about 200 poise and a density of 1 g/cm^3. It is proposed to use a 1:9 scale model for the study.

a. What should be the properties of the simulant in the scale model?

b. What should be the rotation speed of the impeller in the scale model?

c. If we put an encoder to measure the torque on the impeller shaft of the model, how does that scale to the torque on the impeller of the full scale system?

d. In addition to mixing patterns, it is also desirable to look at interfacial transport (a key problem is getting all the water out of their viscous mixture). A colleague proposes to simulate this by looking at interfacial transport of a molecule in the model system. If the diffusion coefficient of water in the full scale system (with its viscous fluid) is 10^{-6} cm^2/s, what should be the diffusion coefficient of the water simulant in the model system to preserve strict dynamic similarity? (Hint: think what ratios should be preserved!)

Problem 3 (10 points): Hydrostatics and Archimedes Law. A hydrometer is a classic tool for measuring the salt concentration in a fluid by determining its density (used in salt water fish tanks, for example). You can make one easily by taking a plastic sphere with a density a bit greater than your target density and gluing an empty straw (air filled, so negligible density) to it. The density of the fluid is determined by dropping the hydrometer into the fluid and seeing how much of the straw sticks out of the water. Here we examine such a system.

a. For an acrylic sphere (density of 1.18 g/cm^3) and radius 1.5cm, what length of straw of radius 0.25cm is necessary to measure the density of water (1.0 g/cm^3)?

b. If our saltwater tank is at the correct salinity, it should have a density of 1.024 g/cm^3. What length of the straw would stick out of the water for this density fluid?

Problem 4 (10 points): The University is installing a hydroelectric generation system on the St. Joseph River at Seitz Park on the East Race (you may have noticed all the construction down there). The amount of power that can be generated depends on the flow rate, the total head (change in elevation) and turbine efficiency. The average flow rate of the river is 3000cfs (cubic feet per second), of which 600cfs in total is reserved for the waterfall, East and West races, and fish ladder (e.g., other purposes, so you have to subtract this off!). The change in elevation is approximately 3m.

a. If the turbine efficiency is 80% (a typical value for this size installation), what is the expected electrical power output of the system in MW? (unit conversion: 1 cubic ft = 28.3 liters). (Hint: think of the available energy/time from the fluid flow!)

b. The current cost of electricity in Indiana is 0.12 $/kwhr. Based on this, what would be the expected value to the university of the electricity produced in one year?