Problem 1. (20 pts) Consider the flow system depicted below. This type of flow is called lid-driven cavity flow, in which the upper plate drives a shear flow from left-to-right, and the wipers (the ends of the box or cavity) induce a pressure gradient in the x-direction which causes a pressure driven back flow. If we focus our attention on the middle of the box (far from either wiper), and if the box is thin, then the flow is approximately unidirectional in the x-direction (e.g., no component in the y or z directions).

a. Write down the Navier-Stokes equation for momentum in the direction of flow for the central region and show which terms are non-zero. Write down the appropriate boundary conditions.

b. Integrate the differential equation and apply the boundary conditions to obtain the velocity profile in terms of the unknown pressure gradient.

c. The presence of the wipers forces the net flow (e.g., the integrated flow rate) in the x-direction to be zero. Use this condition to determine the pressure gradient induced by the wipers.

Problem 2. (20 pts) Consider the arbitrary fluid element depicted below. A fluid with thermal energy per unit volume $\rho C_p T$ (where $T$ is the temperature and $C_p$ is the heat capacity per unit mass) is flowing through the element with velocity $u$. In addition to convective heat transfer, we also have the conductive (diffusive) heat transfer flux, given by Fourier's Law of heat conduction:

$$q = - k \nabla T$$

which is analogous to the shear stress from Newton's law of viscosity. Here $k$ is the thermal conductivity. Given the statement that thermal energy is conserved, derive a microscopic equation governing energy transport in a flowing liquid. Both the velocity $u$ and the temperature $T$ may be functions of position and time, but physical properties such as $k$, $\rho$, and $C_p$ are taken to be constant.
Problem 3. (20 pts) In Senior Laboratory one of the experiments is measuring the drag on a cylinder at high Re. One of the techniques used is called a "wake traverse", in which we measure both the uniform velocity $U$ upstream of the cylinder, and the non-uniform velocity $u(y)$ downstream of the cylinder. We also measure the upstream pressure $p_0$ and the downstream pressure $p_e$. The diagram from the lab manual is given below.

a. Write down in words the integral momentum balance around the control surface depicted above.

b. Given that we can neglect any drag on the side walls of the control surface, develop an explicit integral expression for the drag per unit length (into the paper) $D$ exerted by the fluid on the cylinder in terms of $p_0$, $p_e$, $U$ and $u$. Both $p_0$ and $p_e$ may be taken to be constant over the inlet and outlet, respectively.

Hint: This problem is pretty much identical to the sluice gate problem you solved for homework, except here the upstream and downstream pressures are measured (no gravity), and the downstream velocity profile is non-uniform, and thus must be left in integral form in the momentum balance. In lab it’s this downstream velocity measurement that’s the tricky part of the experiment...
Problem 4. (10 pts) Index notation. Write down the Navier-Stokes equations and the continuity equation for an incompressible fluid in index notation.

Problem 5. (10 pts) Additional Readings / Multimedia CD questions - just list which answer is the correct one!

1. Flow visualization using the dye release technique (dye ejected from a fixed array of nozzles) reveals which of the below for unsteady flows?
   - A. Pathlines
   - B. Streaklines
   - C. Streamlines
   - D. Streamfunctions

2. Order according to the era they worked in (oldest -> most recent):
   - A. G. I. Taylor
   - B. Hero of Alexandria
   - C. Osbourne Reynolds
   - D. Sir Isaac Newton

3. In order for it to move, an oscillating rudder must be operated at:
   - A. Low Reynolds Numbers
   - B. Low Prandtl Numbers
   - C. High Reynolds Numbers
   - D. High Prandtl Numbers

4. Crooke's Radiometer works because of:
   - A. The momentum of light
   - B. Thermal transpiration
   - C. Maxwell said it should
   - D. Hot gas on the black face of the vane

5. Fluidized mixtures of solid and semi-solid fragments, and hot expanding gases which flow down the sides of volcanoes are known as:
   - A. Pyroclastic flows
   - B. Igneous flows
   - C. Sediment flows
   - D. Mudslides