Problem 1. (20 pts) A mass $M$ is sitting on a plate as depicted below, balanced on a vertical jet of water. If the volumetric flow rate of the water jet is $Q$ and the area of the jet nozzle is $A$, we can do a force balance on the plate. Don't forget that the water slows down and spreads out as it travels upwards! For a jet like this, Bernoulli's equation (resulting from a mechanical energy balance in the absence of friction) provides a good relationship between velocity and height:

$$\frac{1}{2} \rho u_1^2 + P_1 + \rho gh_1 = \frac{1}{2} \rho u_2^2 + P_2 + \rho gh_2$$

Given all this, calculate the steady-state height $H$ of the plate, and determine the minimum flow rate for which the plate can be supported.

Problem 2. (20 pts) Starting from the arbitrary stationary control volume depicted below, derive the continuity equation for a compressible fluid.
Problem 3. (20 pts) Earlier this semester we briefly looked at capillary pressure effects, whose magnitudes are characterized by the surface tension \( \sigma \). In this problem we examine the capillary rise of a column of fluid in a narrow 2-D channel. Consider the system depicted below. Fluid enters the thin gap of width \( 2b \). If the fluid wets the surfaces of the channel (basically, if a drop of the fluid would spread out over the material composing the channel surfaces), then the curvature of the interface will cause a capillary pressure drop between the fluid and the air of \( \Delta p_{\text{cap}} = \sigma / b \). Because the pressure in the fluid behind the meniscus is now less than the atmospheric pressure, it sucks fluid up the gap from the reservoir. It is this phenomenon we wish to investigate.

![Diagram of capillary rise](image)

a. Using hydrostatics, determine the steady-state capillary rise in the channel (e.g., the length of the column of liquid when all motion ceases).

b. Now for the transient problem. For a thin gap we can ignore the end effects at the entrance and at the meniscus (e.g., we take the flow to be unidirectional). We can also ignore all inertial terms in the equation of motion, as the Reynolds number is very small. With these approximations, derive a simple differential equation which governs the length of the column of fluid with respect to time. (Hint: think of plane-Poiseuille flow, flow rates, and mass balances)
Problem 4. (10 pts) Index notation. Solve the following problems using index notation:

a. Decompose a general matrix $A_{ij}$ into the sum of a symmetric matrix and an antisymmetric matrix, identifying each.

b. Write down the continuity equation for an incompressible fluid using index notation.

c. The vorticity is defined as the curl of the velocity. Using index notation and symmetry, show that the divergence of the vorticity is zero.

Problem 5. (10 pts) Additional Readings / Multimedia CD questions - just list which answer is the correct one!

1. Flow visualization using the dye release technique (dye ejected from a fixed array of nozzles) reveals which of the below for steady flows?
   - A. Pathlines
   - B. Streaklines
   - C. Streamlines
   - D. All of the above

2. Order according to their date of death (oldest -> most recent):
   - A. G. I. Taylor
   - B. Hero of Alexandria
   - C. Osbourne Reynolds
   - D. James Clerk Maxwell

3. Gimli Air Force Base is in:
   - A. North Dakota
   - B. South Dakota
   - C. Wisconsin
   - D. Canada

4. The most important dimensionless number to keep constant when using a towing tank to study ship hull design is:
   - A. Reynolds Number
   - B. Schmidt Number
   - C. Froude Number
   - D. Prandtl Number

5. The Johnstown disaster was caused by:
   - A. A ruptured dam
   - B. A ruptured tank of molasses
   - C. A pyroclastic flow
   - D. A burst pipe