Problem 1. (20 pts) Consider the trough of width \( W \) which drains through a slit of width \( 2b \) and length (in the flow direction) \( L \) as depicted below. The trough is initially filled with liquid of density \( \rho \) and viscosity \( \mu \) to a height \( H_0 \). You may neglect any variation into the page (e.g., the problem is two-dimensional). The fluid is assumed to be very viscous (e.g., Karo syrup), and thus inertial effects are negligible. Because the slit is much narrower than the trough \( (2b/W << 1) \), the slit dominates viscous resistance to flow (remember the milkshake and straws??). Finally, because the slit is narrow relative to its length \( (2b/L << 1) \), the flow may be considered unidirectional.

a. What is the magnitude of the initial driving force for the drainage?

b. Solve the Navier-Stokes equations for the velocity profile in the slit and the initial drainage rate for fluid draining out of the trough. Assume unidirectional flow!

c. Develop a differential equation describing the height \( H \) of the fluid in the tank as a function of time.
Problem 2. (20 pts) Consider the arbitrary fluid element depicted below. A fluid with thermal energy per unit volume $\rho \, C_p \, T$ (where $T$ is the temperature and $C_p$ is the heat capacity per unit mass) is flowing through the element with velocity $\mathbf{u}$. In addition to convective heat transfer, we also have the conductive (diffusive) heat transfer flux, given by Fourier's Law of heat conduction:

$$ q = -k \nabla T $$

which is analogous to the shear stress from Newton's law of viscosity. Here $k$ is the thermal conductivity. Given the statement that thermal energy is conserved, derive a microscopic equation governing energy transport in a flowing liquid. Both the velocity $\mathbf{u}$ and the temperature $T$ may be functions of position and time, but physical properties such as $k$, $\rho$, and $C_p$ are taken to be constant.

Problem 3. (20 pts) Consider the sprinkler depicted below. The radius of the arms is $R$, the area of the jet outlet is $A$ (for each arm), and the total flow rate of water is $Q$ (e.g., $Q/2$ for each arm). Using an integral momentum balance, determine the torque exerted on the center of the sprinkler if the arms are stationary.
Problem 4. (20 pts) Index notation / Additional Readings / Multimedia CD questions

1. Flow visualization using the time-lapsed photography of a tracer particle technique reveals which of the below for *unsteady* flows?
   A. Pathlines
   B. Streaklines
   C. Streamlines
   D. All of the above

2&3. Match the name up with an item for which they achieved recognition:
   1. Pneumatics
   2. Couette Instability
   3. Stress-Strain Relationship
   4. Turbulent Pipe Flow

   A. Sir Isaac Newton
   B. Hero of Alexandria
   C. Osbourne Reynolds
   D. G. I. Taylor

4. What fraction of an iceberg’s mass is below the waterline when it is floating free in the ocean?
   A. 1/3
   B. 1/2
   C. 3/4
   D. 7/8

5. Using no more than two sentences, describe how temperature inversions have led to acoustic shadows on battlefields.

6. Write down the continuity equation for an incompressible fluid using index notation.

7. Using index notation, write down the representation of the symmetric part of the general matrix $A_{ij}$.

8. The vorticity is defined as the curl of the velocity. Write down this defining relation using index notation.

9. What is the most general isotropic, third order, physical tensor?

10. Write down an example of an isotropic fourth order tensor.