Problem 1. (20 pts) Starting from the arbitrary stationary control volume depicted below, derive the Cauchy stress equations (the momentum equations left in terms of the total stress tensor $\sigma$). Hint: don't forget the volumetric source of momentum (e.g., the body force) due to gravity, and that the surface stress is $\sigma \cdot n$.

Problem 2. (20 pts) Consider the velocity distribution acquired by the slow flow of a very viscous fluid down an incline, as is depicted below. In contrast to the problem considered in class, here we have a breeze trying to blow the fluid back up the incline, in addition to the gravity trying to draw the fluid down. We model the effect of the breeze with a constant shear stress $\tau$ at the upper surface of the fluid (note: $\tau$ will be negative). For some film depth $\delta$, angle $\theta$, density $\rho$ and viscosity $\mu$, how large does this shear stress have to be for the net fluid velocity (e.g., the total flow rate) to be zero? This would be the point where the fluid stops going down the plane (on average, anyway) and starts going back up it. Solve the problem assuming unidirectional flow.
Problem 3. (20 pts) Consider the sprinkler depicted below. The radius of the arms is $R$, the area of the jet outlet is $A$ (for each arm), the density of the fluid is $\rho$, and the total volumetric flow rate of water is $Q$ (e.g., $Q/2$ for each arm). Using an integral momentum balance, determine the torque exerted on the center of the sprinkler if the arms are stationary.

Problem 4. (20 pts) Index notation / Additional Readings

a. (2 pts) Provide two interpretations of $\rho u$

b. (2 pts) The symmetric part of the rate of strain tensor is usually given the symbol $e_{ij}$. What is its representation in terms of $u_i$? (Index notation, please!)

c. (2 pts) Write down the continuity equation for an incompressible fluid using index notation.

d. (2 pts) Match up the kinematic viscosities of the following materials:
   1. Water
   2. Air
   3. Glycerine
   4. Mercury
   A. 0.118 cSt
   B. 1.0 cSt
   C. 17.0 cSt
   D. 650 cSt

e. (2 pts) Crooke's Radiometer works because of:
   A. The momentum of light
   B. Thermal transpiration
   C. Maxwell said it should
   D. Hot gas on the black face of the vane

f. (2 pts) Match the name up with an item for which they achieved recognition:
   1. Couette Instability
   2. Pneumatics
   3. Turbulent Pipe Flow
   4. Stress-Strain Relationship
   A. Hero of Alexandria
   B. Osbourne Reynolds
   C. Sir Isaac Newton
   D. G. I. Taylor
g. (2 pts) Fluidized mixtures of solid and semi-solid fragments, and hot expanding gases which flow down the sides of volcanoes are known as:
   A. Pyroclastic flows
   B. Igneous flows
   C. Sediment flows
   D. Mudslides

h. (6pts) Briefly identify the physical mechanism described by each of the following terms:

1. \( \mu \frac{\partial^2 u_x}{\partial y^2} \)
2. \( \rho \frac{\partial u_i}{\partial t} \)
3. \( \rho \frac{v_r v_0}{r} \)

Extra Credit. For 4 bonus points - do only when you have finished the rest of the exam! Solve for the steady-state angular velocity of the sprinkler in problem 3 if the sprinkler arms are frictionless and free to rotate.