Problem 1). (20 points) Beginning with the statement that mass is a conserved quantity, derive the continuity equation for a time dependent, compressible flow.

Problem 2. (20 points) Consider unidirectional, laminar, steady flow in a channel of width 2 b as depicted below. What is the flow rate per unit extension out of the plane of the paper Q/W as a function of the pressure gradient in the x-direction?
Problem 3. (20 points) Hero's Fountain. In class we demonstrated Hero's Fountain, attributed to Hero of Alexandria a couple of millenia ago. In this problem we analyze its performance.

a). Neglecting all frictional losses, what is the exit velocity and flow rate of the fountain? All pipes are 1cm ID smooth tubes.

b). Modify your answer by accounting for the head losses in the pipes and fittings. Correlations for friction factors in pipes and fittings are given below. You may take the total length of pipe to be 100 cm.

\[ h_L = \frac{(u)^2}{2g} \sum K + 4f_f \frac{L}{D} \frac{(u)^2}{2g} \]

\[ f_f = \begin{cases} \frac{16}{Re} & ; \ Re < 2100 \\ \frac{0.0791}{Re^{1/4}} & ; \ 3000 < Re < 10^5 \end{cases} \]

\[ \frac{1}{\sqrt{f_f}} = 4.0 \log_{10}(Re f_f) - 0.40 \ ; \ Re > 3000 \]

<table>
<thead>
<tr>
<th>Fitting</th>
<th>K value</th>
</tr>
</thead>
<tbody>
<tr>
<td>sudden contraction</td>
<td>0.45</td>
</tr>
<tr>
<td>sudden expansion</td>
<td>1.0</td>
</tr>
<tr>
<td>45° elbow</td>
<td>0.35</td>
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</tbody>
</table>
Problem 4. (20 points) Thermal boundary layers. Consider the system depicted below. The fluid velocity is just unidirectional simple shear flow in the x-direction. The fluid enters with a temperature of zero (e.g., we've already subtracted off some reference temperature), and gets heated by some constant wall heat flux $q_w$. The thermal energy equation is given by:

$$\frac{\partial T}{\partial t} + u \cdot \nabla T = \alpha \nabla^2 T$$

with boundary conditions:

$$q_w = -k \frac{\partial T}{\partial y} \bigg|_{y = 0} \quad , \quad \frac{\partial T}{\partial y} \bigg|_{y = 0} = 0 \quad , \quad T \bigg|_{y \rightarrow \infty} = 0$$

a). Render the equations dimensionless using a length scale $L$ in the x-direction, and determine the conditions under which we can expect a thin boundary layer in the y-direction (e.g., how big does $L$ have to be?). You may assume steady-state, with no variation in the z-direction.

b). Show that the thermal boundary layer equations yield a self-similar solution, obtaining the similarity rule and similarity variable in canonical form. Using this, determine the temperature at the plate as a function of $x$ to within some undetermined multiplicative constant. Note that you don't have to get the transformed ODE or solve it to do this!
Problem 5. (20 points) Pump Curves / Additional Readings:

The first five questions refer to the pump curve on the next page:

1. It is desired to pump 20 liters/sec from a pond to an elevation of 70 meters. If we neglect all frictional losses (say we use a really fat pipe!) is the pump HH80 recommended for the job?

2. What is the RPM required to do the job?

3. What is the work done by the pump on the fluid?

4. What is the efficiency of the pump at the operating conditions?

5. It may be convenient to put the pump part-way up the hill (you don't want it to flood at high pond water levels!). What is the maximum height it can be located at for safe operation? (Note: 1atm = 10.3 m water)

6. Small variations in initial conditions sometimes result in huge, dynamic transformations in concluding events. This statement summarizes the idea behind:
   A. The Peanut Butter Effect
   B. The Butterfly Effect
   C. Poiseuille flow
   D. The Prandtl Number

7. Snapping shrimp produce their snapping sound by closing their snapping claw rapidly. This causes a fast flowing water jet to form, followed by the formation of a bubble, which subsequently collapses. This collapsing bubble creates a shock wave, causing the snapping sound. The formation and collapse of such bubbles is also very important in chemical processing and handling. The formation and collapse of bubbles within a liquid is also known as:
   A. Creeping Flow
   B. Poiseuille Flow
   C. Cavitation
   D. Occulation

8. The properties that give Super Soaker water guns their capacity and range are:
   A. The compressibility of both water and air
   B. The compressibility of water and incompressibility of air
   C. The incompressibility of water and the compressibility of air
   D. The incompressibility of both water and air

9. The purpose of the ridges on Frisbees is to:
   A. Induce Poiseuille Flow
   B. Delay boundary layer separation
   C. Provide gyroscopic stability
   D. Both B and C

10. Ensuring that the NPSHR is met for a pump installation will help prevent:
    A. Cavitation in the pump
    B. Damage to pipes and fittings due to vibrations
    C. Reduced pump performance
    D. All of the above