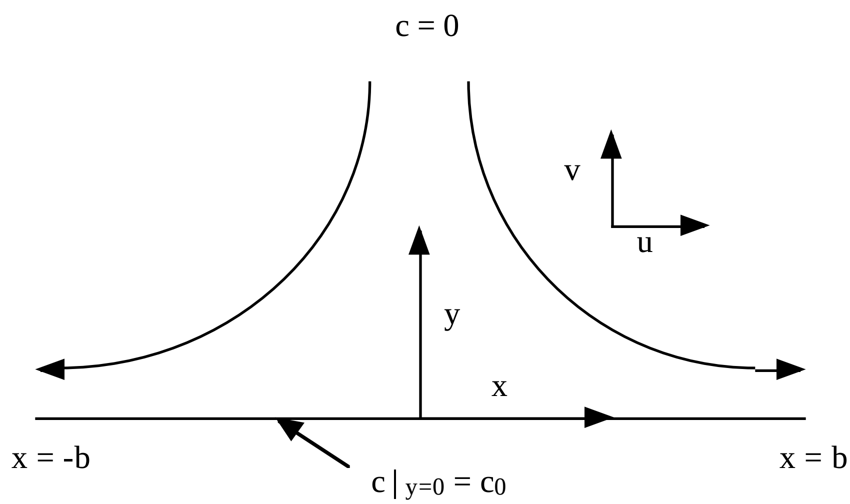
CHEG 355 Transport Phenomena I
Final Exam

December 12, 2005

Closed Books and Notes

Problem 1. (30 points) Scaling analysis of boundary layer flows. An important problem in mass transfer is the rate of diffusion of a solute from a planar surface in the presence of a stagnation flow. Consider the two-dimensional problem depicted below. A planar jet impinges on a plate of width 2b.



a. Very close to the surface the velocity profile is known exactly: for a viscous flow the velocity in the x-direction is given by u = A xy, with a corresponding (non-zero!) velocity in the y-direction. Using the continuity equation and the condition that v vanishes at the plate, determine v.

b. The concentration distribution is given by the convective diffusion equation (both u and v are non-zero!). Scaling this equation, determine the conditions under which we would get a boundary-layer problem for mass transfer (e.g., under which we can ignore diffusion in the x-direction), and the characteristic boundary layer thickness.

$$\underset{\sim}{u} \cdot \underset{\sim}{\nabla} c = D \nabla^2 c$$

c. If the mass flux at the surface is given by:

$$N \Big|_{y=0} = -D \frac{\partial c}{\partial y} \Big|_{y=0}$$

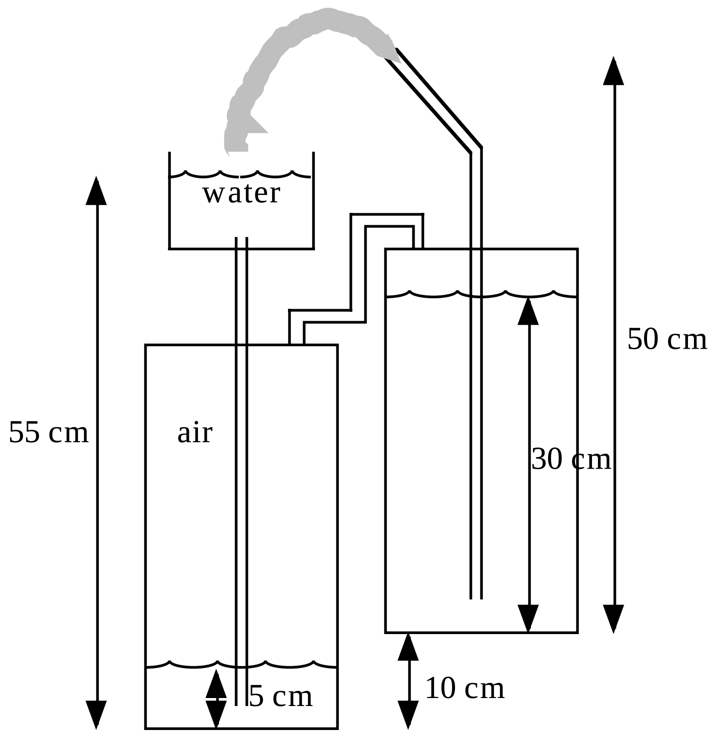
determine via scaling analysis how the total rate of mass transfer from a plate of width 2b in the x direction and W in the third direction (e.g., into the paper, the direction that doesn't matter) scales with the parameters in the problem.

d. Using simple affine stretching, show that the problem admits a self-similar solution. Solve the resulting ODE for the concentration distribution. You may leave your final answer in terms of an integral.

Problem 2. (20 points) Hero's Fountain. In class we demonstrated Hero's Fountain, attributed to Hero of Alexandria a couple of millenia ago. In this problem we analyze its performance.

a). Neglecting all frictional losses, what is the exit velocity and flow rate of the fountain? All pipes are 0.5cm ID smooth tubes.

b). Modify your answer by accounting for the head losses in the pipes and fittings. Correlations for friction factors in pipes and fittings are given below. You may take the total length of pipe to be 100 cm. You will probably need to do a couple of iterations to get the friction factor right.



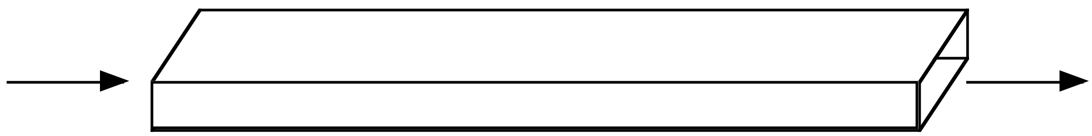
$$h_L = \frac{\langle u \rangle^2}{2g} \sum K + 4 f_f \frac{L}{D} \frac{\langle u \rangle^2}{2g}$$

$$f_f = \frac{16}{Re} \; ; \; \text{Re} < 2100 \qquad f_f \approx \frac{0.0791}{Re^{1/4}} \; ; \; 3000 < \text{Re} < 10^5$$

$$\frac{1}{\sqrt{f_f}} = 4.0 \log_{10}\left(\text{Re}\sqrt{f_f}\right) - 0.40 \; ; \; \text{Re} > 3000$$

Fitting	K value
sudden contraction	0.45
sudden expansion	1.0
45° elbow	0.35

Problem 3). (20 points) Plane Poiseuille Flow: A problem which is currently being investigated in bioengineering laboratories is the phenomenon of cell adhesion to surfaces in the presence of hydrodynamic stresses. This is very important in the design of biocompatible materials, for example. To study this, a researcher has built a rectangular flow cell which is 100μm deep, 2mm wide, and 2cm long. The objective is to have a wall shear stress (e.g., stress at the lower wall - the 2mm x 2cm surface - where cell adhesion is being studied) of 100 dyne/cm^2. If the working fluid has the same viscosity as water, what should the flow rate of the pump supplying the fluid be? Make any simplifications you need to, but provide a **brief** justification.



Problem 4. (30 points) Pump Curves / Additional Readings / Short Answer:

The first five questions refer to the pump curve on the last page:

1. It is desired to pump 60 liters/sec from a pond to an elevation of 30 meters. If we neglect all frictional losses (say we use a really fat pipe!) is the pump CP200 recommended for the job?

2. What is the RPM required to do the job?

3. What is the work done <u>by</u> the pump <u>on</u> the fluid per unit time?

4. What is the efficiency of the pump at the operating conditions?

5. How far up the hill from the level of the pond can we put the pump? (Again, neglect frictional losses) (Note: 1atm ≈ 10.3 m water)

6. In our class demonstration, when we allowed a rod to settle in a viscous liquid (Re = 0), we observed that it didn't rotate. If inertial effects are important, it does. Why is the behavior different in this case?

7. For a shear stress of 100 dynes/cm^2 in the turbulent flow of water through a pipe, about how rough does the pipe wall have to be before it influences the flow?

8. At high Re, drag principally results from:
A. Potential Flow
B. Boundary Layer Separation
C. Skin Friction
D. Turbulence

9. Ensuring that the NPSHR is met for a pump installation will help prevent:
A. Cavitation in the pump
B. Damage to pipes and fittings due to vibrations
C. Reduced pump performance
D. All of the above

10. The displacement thickness is defined as:

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U} \right) dy$$

Provide a brief physical interpretation of this quantity.

11. Just as in the case of turbulent momentum transfer where we define a turbulent kinematic viscosity ν_t, so in the case of turbulent energy transfer we can define a turbulent thermal diffusivity α_t. The turbulent Prandtl number is the ratio of these quantities ($Pr_t = \nu_t / \alpha_t$). What is its approximate magnitude and why?

12. Experimentally, how can you most accurately calculate the shear stress on a plate in boundary layer flow at high Reynolds numbers from the velocity profile?

13. Give a physical description of the Reynolds stress (e.g., where does it come from?).

14. Using index notation, write down the force exerted by the surrounding fluid on an arbitrary object in terms of the stress tensor σ_{ij}.

15. Briefly describe one method for preventing stall on an aircraft wing.

SYKES PUMPS

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DATE OF ISSUE 9 AUGUST 2002

PUMP : CP-200

SUCTION	DISCHARGE	MAX. SPHERE	IMPELLER	IMPELLER	IMPELLER & WEAR PLATES
200mm	200mm	80mm	3 VANE	ø295mm	316 S/S