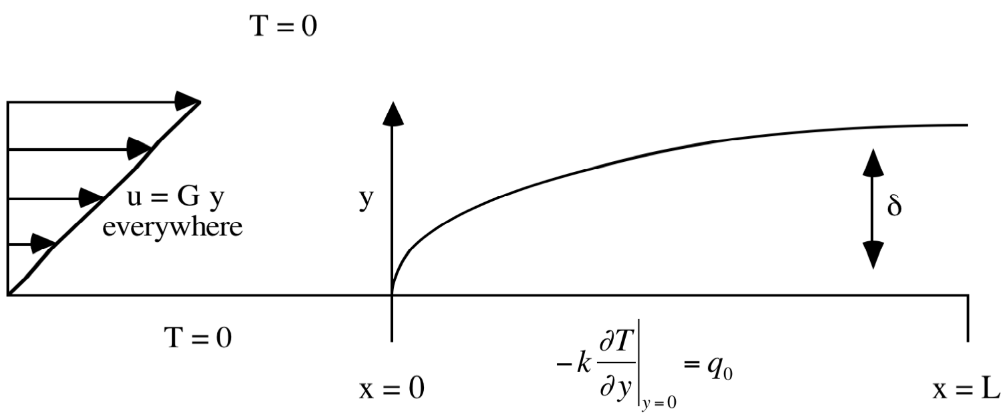
PROBLEM SET 10 CBE 30355 DUE 11/16/23

1. Scaling of the Transport Equations: Energy transport is analogous to momentum transport and the equation governing energy transport may be scaled in the same way as the momentum equations were in class. Consider flow past the flat plate y=0 as depicted below. Fluid with a temperature T = 0 flows along the plane in the x direction with velocity u = Gy where G is the shear rate (e.g., plane Couette flow without the upper plane). The plate is maintained at a temperature T = 0 for x < 0 and a constant heat flux q_0 for x > 0. The governing equations and boundary equations are given below.



$$G\,y\,\frac{\partial T}{\partial x} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$

$$T\Big|_{y=0,\,x<0} = 0 \qquad T\Big|_{y>>0} = 0 \qquad -k\left.\frac{\partial T}{\partial y}\right|_{y=0} = q_0$$

a). Using L as the characteristic length in the x direction and δ the length in the y direction, render the energy transport equations dimensionless. For what value of δ is convection (the left hand side) balanced by a diffusive term (in the right hand side)? Using this scaling, what dimensionless parameter must be small to neglect one of the two diffusion terms (the dimensionless parameter should no longer involve δ)? Doing this, obtain the simplified thermal boundary layer equation for this problem.

b). Using scaling analysis, determine the expected temperature of the plate (e.g., at y = 0) at x = L.

2. A nice demonstration of both boundary layers and inertial secondary currents is the Tea Leaf Problem: as will be demonstrated by your classmates if a pot (or beaker) with tea leaves is rotated then the tea leaves (which tend to settle out on the bottom) are forced to the sides, but when the beaker is stopped the tea leaves all accumulate in a puddle in the center of the bottom. You can see the latter part of the problem in a tea cup after stirring it up and dispersing the leaves as well. Here we examine the mechanism by which a beaker full of water, initially at rest, spins up when the beaker is rotated. We look at the effect of both the side walls of the cylindrical beaker and the bottom.

a. Consider a cylinder of radius R without a bottom where the fluid is initially at rest. At time t=0 we rotate the cylinder with an angular velocity Ω. Eventually the entire fluid in the cylinder will also rotate with this velocity. By scaling the appropriate terms in the equations of motion, *estimate* how long this will take. If R = 5cm and the fluid is water, what is the numerical value of this characteristic time?

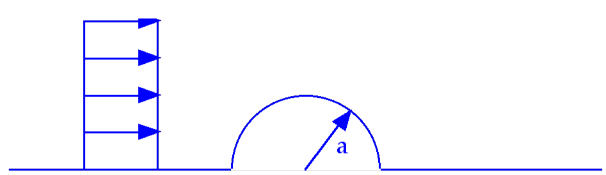
b. Now for the hard part! Let's put a bottom on the cylinder (e.g., now it's a beaker). When we start to rotate the beaker a boundary layer sets up on the bottom, which very quickly approaches steady state. This boundary layer accelerates fluid in the θ direction (via diffusion from the bottom) which is then thrown outwards radially via centrifugal force. By scaling the equations (both r and θ momentum this time!) determine the characteristic radial velocity and the boundary layer thickness.

c. The boundary layer is essentially pumping accelerated fluid into the rest of the beaker. If the fluid height in the beaker is H, determine the characteristic spin up time (e.g., how long it takes the boundary layer to fill up the vessel) as a function of the parameters in the problem.

d. If H is 10cm and Ω is 20 radians/s, what is the numerical value of this spin up time, and how does it compare to the answer in part a?

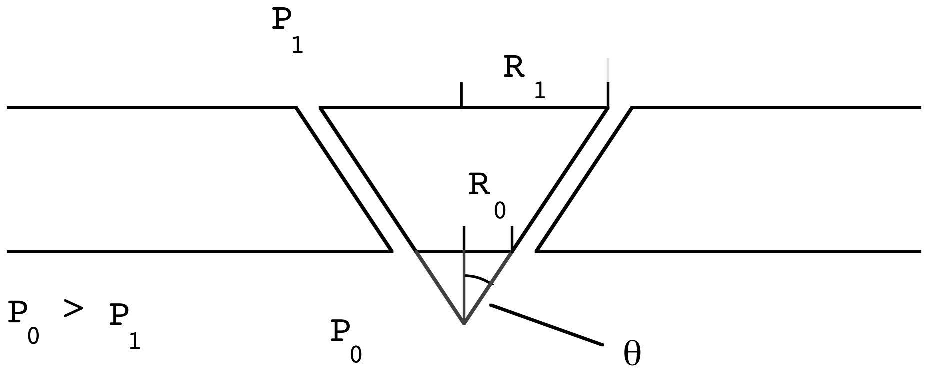
3. Consider uniform, high Re flow past a cylinder of radius a imbedded in a plane as depicted below. Note that we're ignoring any no-slip condition on the plane itself, as well as on the cylinder.

a. For ideal potential flow, calculate the lift and drag on the cylinder. Assume that the pressure inside the cylinder is just the undisturbed p_0 far from the cylinder.



b. Now assume that the flow separates at the top of the cylinder and that there is no pressure recovery from the ideal potential flow solution pressure minimum. What is the magnitude of the lift and drag under these assumptions? Note that the actual lift and drag will be a bit less than these values, but they are reasonable estimates.

4. A conical cork is used to control the flow of air through a conical hole as is depicted below. For what values of R_1 and R_0 will the plug be blown out of the hole? The flow is considered to be ideal and inviscid, and the cork is massless. You should find that the force on the cork is independent of both the conical angle θ and the width of the gap surrounding the cork.



Hint: Assuming parallel flow in the gap, use Bernoulli's equation and continuity to determine the velocity in the gap. To simplify the algebra, use a coordinate system s defined from the imaginary vertex of the cone, calculate P(s), and integrate over s from s_0 to s_1 where $s_0 = R_0/\sin\theta$ and $s_1 = R_1/\sin\theta$.

What you should learn from these problems:

Problem 1: Scaling of the energy equations.

a. This is a key problem demonstrating the parallels between the momentum boundary layers discussed in class with the simpler (because it's linear) thermal boundary layer problem.
b. Practice scaling equations and using them to estimate desired quantities (such as wall temperature in this case) without actually having to -solve- the problem!
c. Demonstration that boundary layer thickness doesn't always scale as $L^{1/2}$!

Problem 2: Analysis of the momentum equations in cylindrical coordinates.

a. This is a *really* nice example of a scaling problem where, again, you can learn a lot about a system (e.g., characteristic times and the dominant physical mechanisms) without actually solving the problem.
b. In part a you are looking at *diffusion lengths* and *diffusion times* for transient problems. This is analogous to pretty much all uni-directional startup (transient) problems.
c. In part b you are determining the scaling of a steady state boundary layer.
d. In c & d you are using the scaling analysis to determine the dominant mechanism for spin up. You can use *exactly* the same procedure (scaling of equations, determination of characteristic times, lengths, etc.) to (relatively) quickly determine what is going on in a problem without having to solve the whole thing completely! This is something I want you to apply in *all* of your engineering problems, or at least where appropriate!

Problem 3: Bernoulli's Equation/Euler flow:

a. This is an example of why you have to tie down your roof in strong winds...
b. Reinforcing the issue of non-ideality in high Re flows (e.g., D'Alembert's Paradox).

Problem 4: Bernoulli's Equation Problem:

a. A demonstration of the unexpected results sometimes obtained by application of Bernoulli's equation to inviscid flow.
b. Demonstration of how different geometries can lead to similar (or in this case identical) results.