PROBLEM SET 2 CBE30355 DUE 9/12/24

1). Compute the viscosity of air from 200ºK to 500ºK and 1 atm pressure. The Chapman-Enskog equation described in chapter 1 of BS&L (eq. 1.4-14) and the data in tables E.1 and E.2 are useful here (note the correlation at the bottom of table E.2)! Compare your result graphically with the data from (the data table is pretty far down the page):

https://www.engineeringtoolbox.com/air-absolute-kinematic-viscosity-d_601.html

2). The natural log of the viscosity of many liquids is approximately quadratic in the inverse of the temperature in ºK (e.g., the equation in chapter 1 of BS&L, with an extra term).

a. Using this, and data from the web page:

https://www.engineeringtoolbox.com/dynamic-viscosity-motor-oils-d_1759.html

determine constants for such a model for the different grades of motor oil (plot the correlations and the data up using a semilog scale). Convert the SI units for viscosity into poise (cgs system). Be sure to label your graph!

b. What are the temperature coefficients for the different grades at 50°C? This is the fractional change in viscosity per degree centigrade, and can be calculated from your fitting parameters.

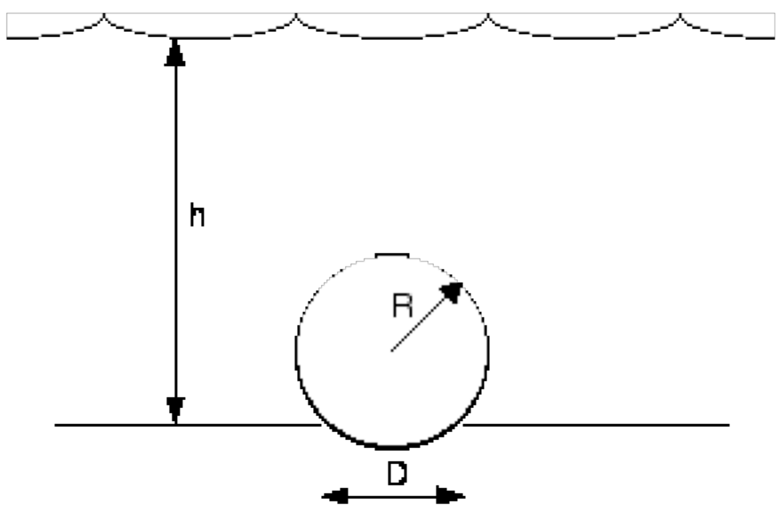
3). Here's a weird application of hydrostatics: Consider a ball of gas (air) floating in space far from any other source of gravity (tidal orbital dynamics would really mess this up!). At the center of the ball, we take the pressure to be 1 atm and the temperature to be a balmy 20°C. As we move outward from the center, the pressure decreases by hydrostatics and the temperature drops by adiabatic expansion (e.g., it obeys both the ideal gas law and PV^{γ} adiabatic expansion). This is the result for a "well mixed" atmosphere, and applies to the earth's atmosphere (at least below the stratosphere, anyway) as well. Our goal is to determine the mass of the ball of gas.

a. Set up the problem as a pair of equations for total mass inside a particular radius and density as a function of position using hydrostatics, the adiabatic gas law, and mass conservation. Don't forget that gravity is a function of position, and that the gravitational attraction *inside* a spherical shell is identically zero!

b. Scale the mass and radius by some unknown values, and then use the equations to determine the characteristic mass and radius (e.g., the magnitude of the scaling parameters so that the resulting dimensionless equations will be of O(1).

c. Solve the dimensionless problem numerically to determine the final value of the mass of gas, and compare it to the mass of the earth. It's pretty easy to set the dimensionless problem up as a pair of coupled first order non-linear ODE's. We will review how to do this in class. Note that the integrand for the change in density is mathematically well-behaved at the origin, however you need to kick it off at a value just above zero (e.g., machine precision) or otherwise fix things to avoid division by zero numerical errors.

4). Pool drains can be dangerous things - there was a tragic case a number of years ago in this area where a child was stuck in a drain on the bottom, plugging it, and drowning as a result. Here we look at a somewhat simpler problem. Suppose a ball of radius R is plugging a drain of diameter D at the bottom of a pool of depth h as depicted above. Obviously, $R > D/2$ or the ball goes down the drain! *Estimate* the conditions under which the net force on the ball is zero for very small ratios of D/R (you can do the *precise* calculation for arbitrary D/R, but the math gets a little messy!). Assume that the pressure distribution in the drain is just atmospheric pressure, and that in the water is governed by the hydrostatic pressure distribution. If R is 0.5 ft and D is 3 inches, what is the corresponding depth? You may neglect the weight of the ball (e.g., its density is very small compared to water).



What you should learn from these problems:

Problem 1: This problem is asked to get you to do several things.

a. To open the book and get practice looking up and using correlations.
b. To recognize that you can calculate the viscosity of air (or any low density gas) from simple collision integral arguments.
c. To get practice cutting and pasting data from charts and graphically comparing data to models/correlations.

Problem 2: Again, several things should be learned:

a. To recognize (and memorize) that for simple fluids such as oils the log of the viscosity can be well correlated with a polynomial in the inverse of the absolute temperature.
b. To refresh your memory on how you do such a linear regression curve fit.
c. To get practice using such a correlation to calculate quantities such as the temperature coefficient.
d. To recognize that, for a viscous liquid, the viscosity changes by about 4% to 7% per degree C – this is a pretty large variation, and is why you can't clean viscous liquids out of a vessel with cold water!
e. To get practice with the mixing rule for simple fluids (doesn't work for more complicated mixtures).

Problem 3: Lots of very important things:

a. To refresh your memory on how to use first year calculus to change/eliminate variables.
b. To combine several equations describing different physical phenomena (mass conservation, hydrostatics, gravitation, adiabatic gas law) together to get a mathematical description of a pretty off-the-wall problem.
c. Your first practice scaling equations to "figure out the answer before you start." This will be a recurring theme in the class, and something you will find useful all your careers.
d. Refresh your memory on how to solve systems of first order ODE's numerically.

Problem 4: More hydrostatics:

a. Connect the mathematics of hydrostatics and buoyancy to a simple problem.
b. Practice making the "flat Earth limit" approximation that is often used to (drastically) simplify calculations.
c. Get a more physical understanding of how pressure distributions are connected to forces.