1). A classic "Honorable Mention" on the Darwin Awards website is the saga of "Lawn Chair Larry" who decided to go flying by attaching 42 helium filled 113 ft^3 weather balloons to his lawn chair. Instead of leveling off at around 30ft of altitude, he wound up at 16,000ft and actually was cited for violating LAX airspace.

a. I'd like you to analyze this problem in hydrostatics and determine if it is possible to control elevation with any precision. Note that in addition to the weight of Larry and his chair (and the balloons, and the helium – you can get a good estimate of the weight of the balloons by assuming that they have a thickness of 0.051mm) Larry was also carrying about a dozen gallon milk jugs full of water. Assuming that he had just one more balloon than necessary to take off, you can get a good estimate of the weight of his entire system, and how much Larry and his chair weighed.

b. Now assuming that the volume of the balloons didn't change with altitude, determine the altitude he would go to with the "extra balloon". Use the "well-mixed atmosphere" assumption (adiabatic expansion) coupled with hydrostatics to get the variation of density and lift with altitude. At least gravity is constant this time!

c. The key error Larry made was ignoring the effect of the elasticity of the balloons. Based on his final altitude of 16,000ft, what were the diameters of the balloons at that altitude?

One (of many) url's for the Lawn Chair Larry story is:

http://www.legacy.com/news/celebrity-deaths/article/larry-walters-the-flying-man

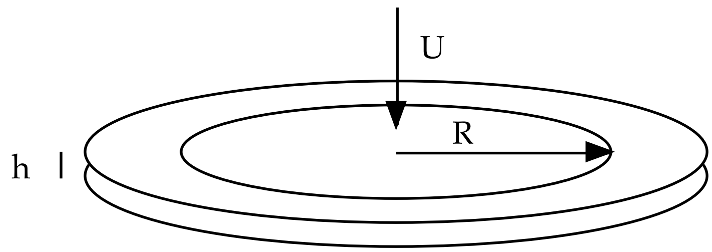
PS: A movie "Danny Deckchair" came out a while back, which is (very) loosely based on Larry's adventure. I've got the DVD, and it's pretty amusing. If you guys want to borrow it (and actually have something that can play a DVD), let me know.

2). The fish tank in my old office had about 50 gallons of water in it. In theory, the water was fresh – but ND tap water actually has quite a bit of salt in it. The water slowly evaporated at a rate of 1 gallon/week, and all the water evaporating was fresh (the salt stays in the tank). We want to see how the salt concentration changed with time under two different scenarios:

a. If I just topped off the water every week with more tap water, how long would it be before the concentration increases by 50% (e.g., 1.5x the original value)? (this is really easy, and pretty much what happened – fortunately goldfish don't mind salty water).

b. You are -supposed- to do weekly partial water changes (I tended to forget, alas). The idea is that you remove some water from the tank, and then replace it with more fresh tap water (after removing the chlorine, of course!). How much water should I have removed/exchanged each week so that the steady-state concentration is no more than twice the tap water concentration? What is the time for it to reach about 90% of this steady-state value (estimate only)? Note that the amount extracted each week is pretty small in comparison to the total volume of the tank, so you can approximate the process as continuous rather than discrete - this makes it much easier!

3). In our first lecture, I demonstrated what happens when a suspension is squeezed between two parallel-plates as is depicted below:



In this case the fluid of volume V is inserted between the plates, and the upper plate falls with a velocity U in the -z direction. The lower plate is fixed, so the gap width h is governed by the simple equation:

$$\frac{dh}{dt} = -U$$

a. As the plates move together, the fluid is squeezed out radially. If the initial separation is h_0, use conservation of mass to determine the radius R of the fluid between the plates as a function of time.

b. The radial velocity will be a function of radial position (it is zero in the center, for example). Using the continuity equation in cylindrical coordinates determine the average radial velocity (averaged over h) as a function of r and time.

c. In the limit of small h/R (such as was used in the demonstration), the velocity is dominated by the radial flow. This is the quasi-parallel approximation that always occurs in lubrication problems, such as we shall examine in detail later this term. For this geometry (and for a Newtonian fluid) the radial velocity is a parabola in z and proportional to a function of r and t (what you got in part b, actually). From the no-slip condition, it is also zero at both z=0 and z=h. Using this information, determine the radial velocity profile as a function of r, z, and t. Later on we'll use the momentum equations to determine pressure inside the fluid and the force required for this motion!

4). Index Notation: Using the concept of symmetry, isotropy and index notation, evaluate the following integrals over a **spherical** (e.g., isotropic) surface of radius a:

a. $\int_{r=a} x_1^2 x_2^2 \, dA$

b. $\int_{r=a} x_1^4 \, dA$

c. $\int_{r=a} x_2^2 \left(x_1^2 - x_2^2\right) dA$

d. $\int_{r=a} \left(x_1^2 + x_2^2 + x_3^2\right) dA$

e. $\int_{r=a} x_1 x_3^2 \, dA$

Hint: The integral $\int_{r=a} x_i x_j x_k x_l \, dA$ is a symmetric, isotropic fourth order tensor…

What you should learn from these problems:

Problem 1: This problem is asked to get you to do several things.

a. To get a bit more practice with hydrostatics, this time for a more terrestrial problem.
b. To think about controllability of a system and how things can go wrong.
c. Because it was a cool story and fun to analyze...

Problem 2: Again, several things should be learned:

a. To set up a time-dependent two-component mass balance.
b. To solve a first order linear ODE (e.g., to memorize the general solution, or to memorize where the solution is written down.
c. To think about the concept of residence time.

Problem 3: This problem is a set up for later stuff in the class and has lots of conceptual aspects:

a. To think about how conservation of mass (or volume, for an incompressible liquid) connects velocities in two directions.
b. To think about how *average velocity, flow rate, and velocity profiles* are connected together in a cylindrical geometry.

Problem 4: Index Notation:

a. To get you to fish out the notes on index notation and study them…
b. To demonstrate that it is *far easier* to solve these sorts of problems using index notation, at least after you are proficient in it!
c. Practice using the concepts of symmetry and isotropy.
d. Simplifying calculations by multiplication with the Kronecker delta function (e.g., turning nasty stuff into simple algebraic equations).