PROBLEM SET 8 CBE 30355 DUE 11/3/22

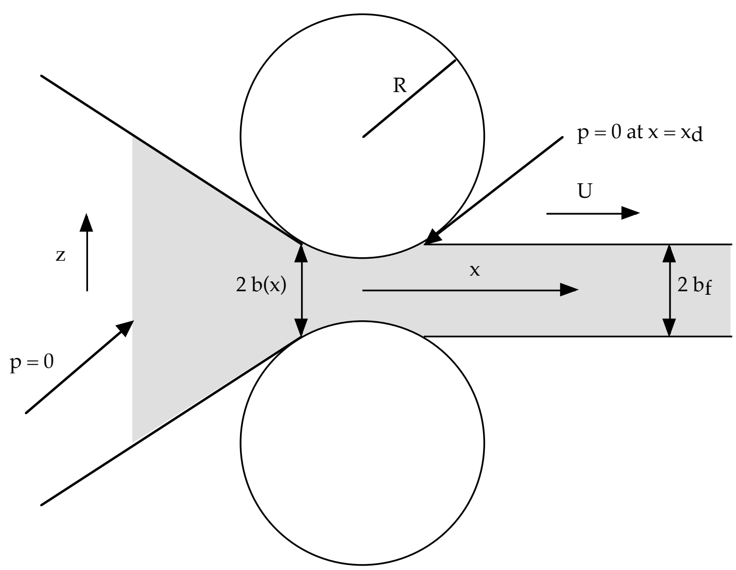
1). Consider the geometry below. Two sheets are drawn with a velocity U between two rollers of radius R whose surfaces are separated by a distance 2 b_0. The idea is to fill the space between the sheets with a viscous fluid of viscosity μ, and all the action takes place in a lubrication layer between the rollers, at least in the limit $b_0/R << 1$. Here we analyze this problem.

a. By scaling the flow equations in the gap (use Cartesian coordinates!), show how the force/width on the rollers F/L scales with the separation distance between the rollers and the other parameters of the problem.

b. The sheet sandwich detaches from the rollers downstream at a point x_d and final gap width $2\, b_f > 2\, b_0$ when the pressure again returns to zero. Develop an implicit integral relation for the ratio b_f/b_0, and show that it doesn't depend on any of the other parameters of the problem.

c. Develop an integral relationship for the dimensionless pressure and force and evaluate it numerically.

Hint: the gap geometry in the lubrication limit is given by $b = b_0 + \frac{1}{2}\frac{x^2}{R}$ where x is the distance along the gap from the point of minimum separation. By continuity, the flow rate per extension into the paper (Q/W) through any vertical plane -must- be the same, and must equal the exit flow rate U*2b_f.



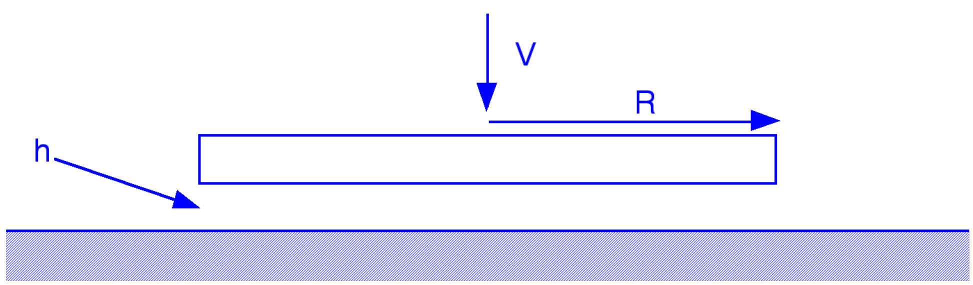
2). In an interesting paper (Davies & Stokes, J. Rheol 2005) a fundamental problem with current parallel-plate viscometers was identified. As you know, the viscosity of a fluid is measured in a parallel-plate viscometer by rotating one plate at a known angular velocity and measuring the resultant torque on the other plate. The viscosity calculation, however, requires accurate knowledge of the gap width between the plates. In a modern instrument it is possible to set the gap accurately automatically -provided- you have the reference point of where the two plates are in contact: e.g., where the gap is zero. For an instrument such as this, you just push a button, the upper plate descends at a constant velocity of 100μm/s, and the gap is considered to be zero when the measured upward thrust on the upper plate exceeds 10^4 dynes, providing the reference point for all subsequent gaps - very convenient! This would be fine if there were no fluid in the gap (e.g., a vacuum), but as you know the viscosity of air is non-zero ($1.8\text{x}10^{-4}$ poise). In this problem we analyze what the effect of air is on the gap zeroing of 3cm radius parallel-plates.

a. For a normal force detection threshold F, approach velocity V, air viscosity μ, and plate radius R, calculate the error in the gap set zero.

b. For the numbers given above, calculate the numerical value of the error in microns. Is it practical to reduce this error by an order of magnitude by decreasing V? Be specific!

c. If we set the gap to 250μm using this incorrect zero (e.g., actual gap differs from 250μm due to the zero error), what is the corresponding error in the viscosity measurement μ_{exp}/μ of a fluid?

d. How can you use the output of the instrument to get the correct viscosity? Be specific! (Hint: It will require analysis of a couple of measurements.)



3). Thermodynamics and Scaling Analysis: In this problem we estimate the temperature and fluid velocity in a chimney as a function of various physical parameters. The chimney is of cross-sectional area A and height H, thus the total volume of air in the chimney is AH. We have a source of energy at the bottom given by Q (assumed to be distributed over the whole area and, unlike a real wood-burning fireplace, to be independent of the gas flow rate). The energy, of course, raises the temperature of the air, causing it to expand. Using the principles of conservation of energy and momentum, and neglecting all frictional and heat losses, we want to estimate the mass flow rate and temperature of the air in the chimney.

a. For “small” values of Q the temperature change (and thus density change) of the air is small. What is the mass flow rate and chimney temperature in this limit?

b. For “large” values of Q the density of the gas in the chimney is much less than that of the surrounding gas. What is the mass flow rate and temperature in this limit?

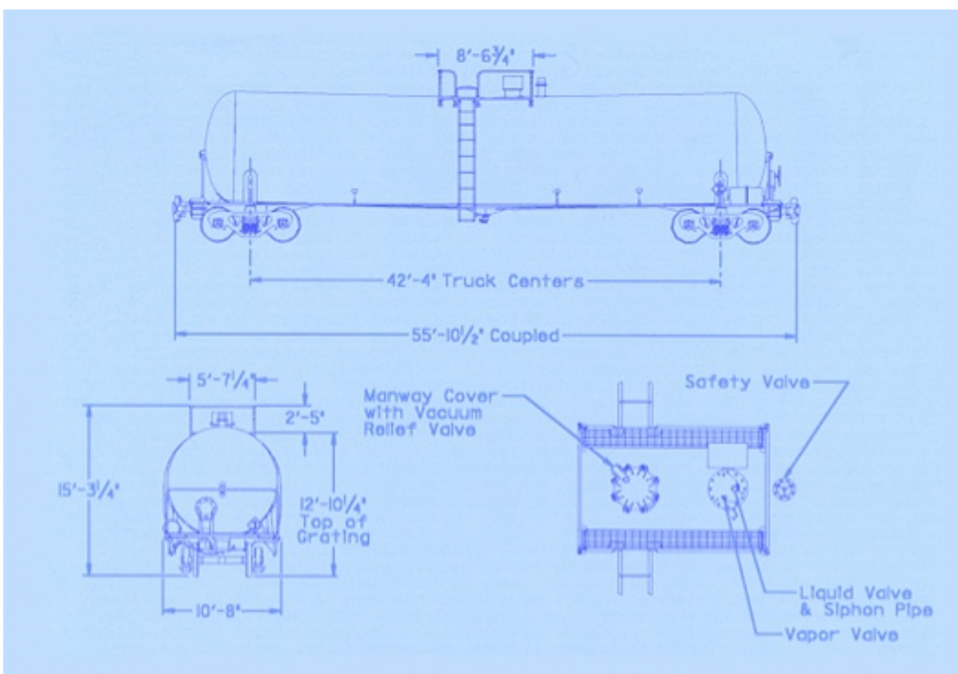
Hint: You are going to have to remember some of your Thermodynamics from last term to get this one!

4). Dimensional Analysis: Modeling air entrainment in a draining tank. The use of rail cars to ship heavy crude down from Canada or from the fields of North Dakota has exploded (sometimes literally, alas) in recent years. As a consequence of safety requirements, tank cars are being redesigned. You are assigned the task of performing a scale model test to determine the maximum drainage rate.

Consider the liquid tanker rail car depicted below. The tanker contains heavy crude with viscosity of 10 poise and density of 0.922 g/cm^3. As the tank drains a vortex will form and eventually air will be entrained into the drainpipe - something we want to avoid. We want to determine the allowable operating conditions for our large tank by studying the behavior of a model system (geometrically similar), but using water as the working fluid.

a. For strict dynamic similarity, what should be the scale down ratio of the scale model?

b. How does the draw off rate (volumetric flow rate) scale between the model and full size tanker?



What you should learn from these problems:

Problem 1: Lubrication: defined flow rate.

a. This is a very nice example of a defined flow rate lubrication problem, although you will need to be careful in applying your boundary conditions!
b. Reinforces that, as in all lubrication problems, the velocity profile across the thin direction is just a parabola… Thus, if you get the three constants by applying boundary conditions, symmetry, etc., the problem becomes *much* simpler!
c. More practice rendering equations dimensionless!

Problem 3: Lubrication flow between two disks:

a. This makes you work through the notes on *defined flow rate* lubrication problems.
b. Demonstrating how you can use the results of one problem to provide information on a rather different one (e.g., tying the results together).
c. Recognizing that sometimes "push button instruments" can yield systematic errors due to unanticipated effects, and thinking through how you could fix the results by proper recognition of *why* the results were in error…

Problem 3: Scaling & Thermodynamics:

a. This problem is a useful example of applying your intuition to get insight into how a real problem (e.g., a chimney) depends on various parameters.
b. Helps to see how thermo and transport are interrelated.

Problem 4: Dimensional Analysis:

a. Another example of scaling, this time using strict dynamic similarity.
b. Application of scaling to a real-world modeling problem.