A Rainbow of Skittles

Materials

- 1. A white or glass plate, slightly depressed in the center
- 2. A bag of Skittles
- 3. A pushpin and a bit of carpet tape
- 4. A 400ml beaker or crystallizing dish
- 5. Distilled water

A classic children's demonstration is the pretty pattern that you can form from brightly colored Skittles dissolving on a plate. There are innumerable YouTube videos of the phenomenon. The procedure is simple: arrange the Skittles around the edges of the plate (with an appropriate color pattern) and then slowly add water to the center of the plate. As the water reaches the Skittles, they begin to dissolve, releasing their sugar and dye into the liquid. These form bands that flow toward the center of the plate.

The explanation of the effect is usually miss-described on the web as being solely due to diffusion. In fact, for this problem diffusion is principally important only in a thin boundary layer adjacent to the candies. The correct explanation is that as the candies dissolve, releasing sugar into the water, the local density increases. This produces a buoyancy driven flow both around the candies and towards the center of the plate. The dye (which is at a concentration too low to affect the buoyancy) simply travels along with the sugar laden fluid. The bands remain separate because diffusion over the length scale of the band width is very slow: diffusion only plays a role over very short length scales in this problem.

There are a number of different demonstrations which can be done to illustrate this effect. Many plates are actually slightly concave up in the center. For such a plate the pattern initially forms regularly (in the downward sloped region) and then as it reaches near the center becomes unstable (and the motion greatly slows down). If the plate is inverted, the center is now convex and the pattern remains regular to the center and proceeds much more rapidly. A flat dish (such as a petri dish, below) also yields a regular pattern. A purely diffusive process would be relatively unaffected by such slight changes in geometry.

The dynamics of the dissolution process can be imaged by looking at a single skittle after immersion. This is easily done by pinning a Skittle with a pushpin and attaching it to the bottom of a crystallizing dish (useful because of the larger aspect ratio and flat

bottom) with a bit of carpet tape. Although it does not appear to greatly affect the dissolution process, fracture of the hard shell can be avoided by briefly heating the tip of the pushpin in a candle flame before insertion. The pinned Skittle is covered with an inverted small beaker, distilled water is added to the crystallizing dish sufficient to cover the candy, and the small beaker is removed. This procedure is desirable to allow the initial swirls from adding the water to damp out as much as possible. The dissolving layer is imaged from the side showing the flow of the dye around the Skittle, followed by the clear sugar solution after the dye coat is removed. The dissolving clear sugar is visible via refractive index variations showing the flow pattern. Distilled water is used as tap water (or at least that at Notre Dame) causes micro bubbles to form once the sugar coat is breached.



Quantitative calculations are also of use. The rate with which the Skittles dissolve is quite closely related to the classic problem of a dissolving spherical particle, important in pharmaceutical applications among others (e.g., Assunção et al., 2024). If the dissolution at the surface is "fast" (not kinetically limited), the solute is dilute, and the viscosity of the fluid is not significantly affected by the dissolving material, then this problem is actually identical to heat transfer from a heated sphere into a quiescent fluid. In that problem the Nusselt number (dimensionless heat transfer coefficient) is a function of the Rayleigh number and the Prandtl number. For mass transfer the Sherwood number (dimensionless mass transfer coefficient) is the same function of the Rayleigh number analog $Ra_s = \frac{D^3g\Delta\rho}{D_s\mu}$ (where the thermal expansion driving force $\rho\beta\Delta T$ is replaced by $\Delta\rho$ (the difference between the solution density at the surface from that far away) and the thermal diffusivity is replaced by the molecular diffusivity D_s) and the Schmidt number $Sc = \nu/D_s$ rather than the Prandtl number (Churchill, 1983).

$$Sh = \frac{k_m D}{D_s} = 2 + \frac{0.589 \left[\frac{D^3 g \Delta \rho}{D_s \mu}\right]^{1/4}}{\left(1 + \left(\frac{0.43}{S_c}\right)^{9/16}\right)^{4/9}}$$

Application of this to the dissolution of sugar is complicated by the very high solubility of sugar in water (about 200g of sugar in 100g of water), so it is hardly dilute. Finite concentration effects have been shown to modify mass transfer rates due to convection normal to the surface in the boundary layer (e.g., Acrivos, 1962). At high concentrations the viscosity is greatly increased as well, with the diffusivity similarly decreasing. For purposes of estimation, we shall take the surface concentration to be about 20% (the point where viscosity and diffusivity begin to diverge significantly from the dilute result) yielding a density difference at the surface of 0.08 g/cm^3 . The diffusivity of sugar in water is $5x10^{-6}$ cm²/s, so the Schmidt number is about 2000 for a dilute solution. Since the laminar momentum and mass transfer boundary layers scale as the square root of Sc, the velocity profile is some 40 times thicker than the solute profile most of the convected fluid is actually sugar and dye free for laminar flow. For a candy with diameter of 1cm, the modified Rayleigh number is approximately 10⁹, close to the point where the flow transitions from laminar to turbulent (e.g., Maestre, et al., 2021). Inserting these values into the Churchill correlation for the Sherwood number yields Sh $\sim 10^2$, so the steady-state mass transfer rate is some 50 times what would be expected from diffusion alone.



Quantitative application to dissolving Skittles is even more challenging. The candy is an oblate spheroid with major diameter of 1.27cm and minor diameter of 0.85cm. It consists of a thin hard sugar shell (approximately 700μ m thick) surrounding the chewy core. The dye itself is confined to an even thinner layer on the surface of the shell, and there is a further insoluble protective coating on top of this on which the "S" is printed. The compositions and thicknesses of these layers are unfortunately proprietary. If a Skittle is simply dropped into a beaker of water, the dyed layer appears to dissolve away in about a minute, with a visible downward flow at the base of the candy and the removed dye confined to a thin layer on the bottom of the beaker.

More specific information about the dissolution process may be obtained from close examination of a Skittle supported by a pushpin away from the bottom of the crystallizing dish. Within 6 seconds of immersion the protective outer coat is disrupted, and after about 30s the insoluble coat has completely sloughed off. This is followed by dispersion of the dye and dissolution of the hard sugar shell. From images it is apparent that the Rayleigh number is indeed high enough to induce fluid instabilities in the wake region. The observed behavior and pattern is remarkably similar to studies of boundary layer separation and instability for natural convection around spheres (e.g., Schütz, 1963; Kitamura et al., 2015; and Lee & Chung, 2017). As in these experiments, at high Rayleigh numbers the boundary layer separates below the equator and the separated layer breaks into vortices which significantly increase the mass transfer coefficient in the wake region. For the Skittles, the time for the dye to locally vanish is roughly inversely proportional to the local mass transfer coefficient. Examination of the images suggests that the mass transfer coefficient is highest in the wake region and a minimum (longest dye persistence) near the separation region just below the equator in agreement with the measurements of Schütz at comparable Ra. While complicated by the sloughing of the protective coat, it appears that the ratio of the mass transfer coefficient in the wake region to that just below the equator is approximately a factor of 2. Comparison of this ratio to the measurements of Schütz for mass transfer from a sphere suggests that the equivalent Rayleigh number is approximately 1.5x10⁹.

The separated sheet of dense, sugar laden fluid is unstable to a transverse disturbance with a wavelength of approximately 500μ m. These vortices then form rivulets which further interact in the wake region and give rise to the higher mass transfer rate at the bottom (where the dye first disappears). Although more difficult to see, the rivulets persist in number and location even after the dye coat has completely vanished. While the rivulet formation mechanism and wavelength selection is less well described in the literature, it appears that the breakup of the sheet is a gravitational Rayleigh-Taylor type instability (e.g., Limat et al., 1992) rather than a Kelvin-Helmholtz shear instability, as that would lead to a vorticity axis perpendicular to the flow. The pattern formed closely matches what was observed by Kitamura et al. (2015) for heat transfer in water at a Rayleigh number of $1.7x10^9$ (e.g., Figure 2.d of that paper). It is remarkable that this simple demonstration can qualitatively (and semi-quantitatively) capture such a complex phenomenon. In the class demonstration the dissolution of the Skittle is projected onto the screen and the observed behavior is compared to the flow pattern in Figure 2 of Kitamura and the dye disappearance rate to the spatially dependent Sherwood number measurements in Figure 6 of Schütz.

Whether the observed flow of the dye away from the Skittles on a plate is solely due to its own gravitational source of momentum or whether the subsequently dissolving sugar applies an additional stress to the surface of the dyed layer, dragging it towards the center of the plate or simply displaces it, is unclear as both processes likely play a role. In either case, however, the dye motion and pattern formation are primarily due to gravitationally driven flow rather than molecular diffusion as is often asserted. Diffusion is unimportant on the length scale of the Skittles for the short duration of the phenomenon.

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