A FIRST COURSE IN
AIR FLOW

by

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1. A BRIEF INTRODUCTION TO AIR FLOW

The concept of continuity and Bernoulli's equation are fundamental to an elementary understanding of air flow, and these matters are dealt with in every textbook on fluid mechanics, hydraulics or aerodynamics. For the sake of completeness and to introduce some of the terms used in subsequent chapters, a brief outline is given here.

The fluid is considered as a continuous medium, so that if we fix attention at one point of the space in which a fluid is in motion, we can observe fluid streaming continuously through the point, and the fluid velocity at the point may conveniently be represented in magnitude and direction by a vector at the point. This is conveniently called the velocity vector, and the

![Local velocity vector and Streamline]

Fig. 1.1. Definition of a Fluid Streamline

whole motion is described by the set of velocity vectors at all points of the space i.e. by the velocity field. If we now draw a line in the fluid so that at every point of the line the tangent is in the direction of the velocity vector at the point, we have a streamline of the fluid, as illustrated in fig. 1.1.

Bernoulli's equation deals with the case where the velocity field is steady, by which we mean that the velocity at each point of the motion does not vary with time.

Consider now a bundle of streamlines as shown in fig. 1.2. Since the

![Individual streamlines]

Fig. 1.2. A Streamtube
direction of the fluid velocity at each point in the surface defined by the streamlines lies along the surface, no fluid crosses it and the fluid contained inside the bundle may be thought of as flowing in a tube, a so-called streamtube.

Consider a section across a streamtube of infinitesimal area \( \delta A \). If the velocity along the tube at this section is \( u \), then in unit time the volume of fluid which crosses across the section is \( u \delta A \). The mass of this fluid is \( \rho u \delta A \). This is the rate of mass flow \( \delta \dot{m} \) across the section, and since no fluid crosses the walls of the streamtube,

\[
\delta \dot{m} = \rho u \delta A = \text{constant along the infinitesimal streamtube} \quad (1.1)
\]

This is one form of the equation of continuity of flow. Various other forms may be derived as shown below. If the streamtube is of finite cross-sectional area, then the rate of mass flow is

\[
\dot{m} = \int_A \rho u dA = \text{constant along the streamtube} \quad (1.2)
\]

where the integration is taken over the area \( A \) of the cross-section. For the particular case in which \( \rho \) and \( u \) are both constant over the sectional area,

\[
\dot{m} = \rho u A = \text{constant along the streamtube} \quad (1.3)
\]

It is sometimes convenient to deal in terms of volume flow rate. For the infinitesimal tube the volume \( \delta Q \) crossing the elementary area \( \delta A \) per unit time is

\[
\delta Q = u \delta A \quad (1.4)
\]

and this is constant along the streamtube only if \( \rho \) is constant along it. For a streamtube of finite cross-sectional area \( A \) the volume flow rate is

\[
Q = \int_A u dA \quad (1.5)
\]

and for the particular case of constant velocity over the section

\[
Q = uA \quad (1.6)
\]

Again, \( Q \) is constant along the streamtube only if \( \rho \) is constant along it.

Consider now steady motion of a fluid along an elementary streamtube. Fig. 1.3 shows an element of length \( \delta s \) and the forces acting on it.

The pressure rises from \( p \) to \( p + \frac{dp}{ds} \delta s \) along the element, and the cross-sectional area increases from \( A \) to \( A + \frac{dA}{ds} \delta s \). The forces due to pressure on the element are

- on the section at \( s \):
  \[ pA \text{ in the s-direction} \]
- on the section at \( s + \delta s \):
  \[ -(p + \frac{dp}{ds} \delta s) \left( A + \frac{dA}{ds} \delta s \right) \text{ in the s-direction} \]
- on the wall of the streamtube: the pressure varies from \( p \) to \( p + \frac{dp}{ds} \delta s \), and the projected area of the wall on a plane normal to the s-direction is \( \left( A + \frac{dA}{ds} \delta s \right) - A \), viz. \( \frac{dA}{ds} \delta s \). So the component of force in the s-direction lies between
  \[ p \frac{dA}{ds} \delta s \text{ and } \left( p + \frac{dp}{ds} \delta s \right) \frac{dA}{ds} \delta s \]

To first order of infinitesimal quantities this is

\[ \frac{dA}{ds} \delta s \]
he net force in the s-direction due to pressure is thus

\[ pA - \left( p + \frac{dp}{ds} \right) \left( A + \frac{dA}{ds} \delta s \right) + pA \frac{dA}{ds} \delta s \]

which reduces simply to

\[ -A \frac{dp}{ds} \]

first order of infinitesimals. The force due to the weight of the fluid is

\[ \rho gA \delta s \]

acting downwards, and if the z-direction is taken vertically upwards, this is

\[ -\rho gA \delta s \]

in the z-direction. The component of this in the s-direction is

\[ -\rho gA \frac{dz}{ds} \delta s \]

the mass of fluid within the element is \( pA \delta s \). The s-component of fluid acceleration along the streamline may be derived by considering the velocity change \( \delta u \) over the length \( \delta s \) of the element, which is

\[ \delta u = \frac{du}{ds} \delta s \]

the time \( \delta t \) in which this velocity change takes place is the time required for fluid to travel the distance \( \delta s \) from one end, where the speed is \( u \), to the other, where the speed is \( u + \frac{du}{ds} \delta s \).

so, to first order of small quantities, the time \( \delta t \) is

\[ \delta t = \frac{1}{u} \delta s \]

the s-component of acceleration \( a_s \) is therefore

\[ \frac{a_s}{u} = \frac{\delta u}{\delta t} = \frac{du}{ds} \frac{1}{u} \delta s \]

and

\[ a_s = \frac{du}{ds} \frac{1}{u} \delta s \]

Equating the mass-acceleration of the fluid to net force, the equation of motion in the s-direction is therefore

\[ \rho A \delta s \frac{du}{ds} = -A \frac{dp}{ds} \delta s - \rho g A \frac{dz}{ds} \delta s \]

which leads to the result

\[ \frac{du}{ds} + \frac{1}{\rho} \frac{dp}{ds} + g \frac{dz}{ds} = 0 \]  \hspace{1cm} (1.7)

Provided that \( \rho \) is constant, this equation may be integrated to give

\[ \frac{\gamma u^2}{\rho} + \frac{P}{\rho} + gz = E \]  \hspace{1cm} (1.8)

where \( E \) is a constant. Multiplication by \( \rho \) gives

\[ \gamma \rho u^2 + \frac{P}{\rho} + \rho gz = P \]  \hspace{1cm} (1.9)

and division by \( g \) gives

\[ \frac{u^2}{2g} + \frac{P}{\rho g} + z = H \]  \hspace{1cm} (1.10)

both of which are forms of Bernoulli's equation. The constant \( P \) is called the total pressure and \( H \) the total head. It should be noted that the result has been derived for steady motion of a fluid under pressure and gravity forces - shear force due to viscous action on the wall of the streamtube has been neglected - and that the density has been assumed constant viz. the fluid has been assumed incompressible. Note also that the integration has been with respect to \( s \), i.e. in the s-direction, along the streamline. The result may thus be stated in words: -

"The total pressure (or total head) is constant along a streamline in steady motion of an inviscid, incompressible fluid".

The equation says nothing about the way that the total pressure changes from one streamline to another.

The first term in equation (1.9) is dependent on fluid velocity \( u \), so we refer to

\[ \frac{1}{2} \gamma pu^2 \]

as the velocity pressure or *dynamic* pressure.

The remaining terms depend on pressure \( p \) and elevation \( z \), and we refer to

\[ p + \rho gz \]

as the *piezometric* pressure.
When the working fluid is air, the static pressure \( p \) is much more important than \( \rho g z \). We shall see that changes in \( p \) in the experiments described later are typically about 1000 N/m\(^2\). For a change of elevation of 1 m, the corresponding change in \( \rho g z \) is about 10 N/m\(^2\). So in practice, Bernouilli's equation for air is frequently written

\[
\frac{1}{2} \rho u^2 + \rho = P
\]  

(1.11)

since the contribution of \( \rho g z \) is usually negligible. The equation is strictly valid if \( p \) is now taken to indicate piezometric pressure.

We have seen that Bernouilli's equation applies only to a fluid of constant density, and it has been mentioned in the previous paragraph that it may be applied to air, for which the density clearly changes with temperature and pressure. Is it possible to make the constant density assumption for air? By an analysis which is beyond the present scope, it may be shown that \( p \) generally exceeds \( \frac{1}{2} \rho u^2 + \rho \) by an amount which increases as the velocity increases. The governing parameter is the Mach number \( Ma \) defined as the ratio of velocity \( u \) to the local velocity of sound \( a \), viz.

\[
Ma = \frac{u}{a}
\]  

(1.12)

Some numerical values, calculated for air flow at 15°C, for which the value of \( a \) is 340 m/s, are shown in Table 1.1. Now according to equation (1.11), the value of the quantity tabulated in the last column of the table should be exactly 1, so the amount by which the numbers in this column exceed unity may be taken as an indication of the error in equation (1.11) due to the compressibility of the air. In the experiments described in the following chapters, the air velocity rarely exceeds about 50 m/s, so the compressibility error is not more than \( \frac{1}{2} \%), and the flow may justifiably be regarded as incompressible.

<table>
<thead>
<tr>
<th>( u ) (m/s)</th>
<th>( Ma = u/a )</th>
<th>( (P - \rho) / \frac{1}{2} \rho u^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.147</td>
<td>1.005</td>
</tr>
<tr>
<td>100</td>
<td>0.294</td>
<td>1.022</td>
</tr>
<tr>
<td>200</td>
<td>0.588</td>
<td>1.089</td>
</tr>
<tr>
<td>300</td>
<td>0.882</td>
<td>1.210</td>
</tr>
<tr>
<td>340</td>
<td>1</td>
<td>1.276</td>
</tr>
</tbody>
</table>

Table 1.1 Calculated difference between total and static pressure as a function of Mach number.

The air density \( \rho \) may be calculated from the barometric pressure \( p \) and temperature \( T \) from the gas equation

\[
\frac{p}{\rho} = RT
\]  

(1.13)

in which the value of the gas constant \( R \) for dry air is

\[
R = 287.2 \text{ J/kg K}
\]  

(1.14)

or

\[
R = 287.2 \text{ Nm/kg K}
\]  

(1.14a)

\( J \) indicates the SI unit of energy, the joule (which is identical with the newton-metre or N m) and \( K \) indicates the unit of temperature, the kelvin. If \( T \) represents temperature in °C, then

\[
T = t + 273.15
\]  

(1.15)

and from equations (1.13), (1.14) and (1.15)

\[
\rho = \frac{p}{287.2 (t + 273.15)} \text{ kg/m}^3
\]  

(1.16)

In this equation, \( p \) is expressed in N/m\(^2\) and \( t \) in °C. Some typical values of \( \rho \) are given in Table 1.2.

<table>
<thead>
<tr>
<th>( t ) (°C)</th>
<th>( \times 10^{-3} ) p (N/m(^2))</th>
<th>0.95</th>
<th>1.00</th>
<th>1.01325</th>
<th>1.05</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.168</td>
<td>1.230</td>
<td>1.246</td>
<td>1.291</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>1.148</td>
<td>1.208</td>
<td>1.224</td>
<td>1.269</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>1.128</td>
<td>1.188</td>
<td>1.203</td>
<td>1.247</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>1.109</td>
<td>1.166</td>
<td>1.183</td>
<td>1.226</td>
<td></td>
</tr>
</tbody>
</table>

Table 1.2. Density of dry air, in units of kg/m\(^3\)

Note that the pressure 1.01325 x 10\(^5\) N/m\(^2\) is the pressure of a standard atmosphere.

In the event of the barometric pressure being given in mm of mercury, this may be converted to units of N/m\(^2\) by the hydrostatic relationship

\[
p = \rho gh
\]  

(1.17)
which gives the pressure \( p \) due to a height \( h \) of liquid of density \( p \) acted on by gravity \( g \). This leads to

\[
1 \text{ mm Hg} = 133.3 \text{ N/m}^2
\]  
(1.18)

Many manometers in general use are calibrated in units of mm of water gauge rather than N/m\(^2\), so it is necessary to make the conversion by the use of

\[
1 \text{ mm water} = 9.807 \text{ N/m}^2
\]

(These conversions from heights of liquid columns to pressure are strictly valid only under conditions of standard gravity).

If the velocity pressure of an air stream of standard density \( (\rho = 1.224 \text{ kg/m}^3) \) is expressed in the form \( h \) mm water gauge, then by expressing \( h \) in units of N/m\(^2\),

\[
9.807h = \frac{1}{2} \times 1.224 \times u^2
\]
or

\[
u = 4.00\sqrt{h} \text{ m/s}
\]  
(1.19)

Alcohol is frequently used as a manometer fluid and the scale is usually compensated inversely to the relative density so that it reads mm water gauge directly. Consider a manometer filled with alcohol of relative density 0.784 (viz. density 1000 \( \times \) 0.784 kg/m\(^3\)) which is required to indicate in N/m\(^2\). If the physical separation of divisions representing increments of 1 N/m\(^2\) is \( s \) m, then expressing pressures in units of N/m\(^2\) gives

\[
1000 \times 0.784 \times 9.807 \times s = 1
\]

\[
\therefore \quad s = 1.301 \times 10^{-4} \text{ m} = 0.1301 \text{ mm}
\]
2. THE AIR FLOW BENCH

Many readers will know that much of the research and development work in aerodynamics is done in wind tunnels, which provide the controlled flow of air required for tests. Some of these tunnels are large and complex, representing major engineering achievement in their own right, and requiring considerable power to operate. The air flow bench is in the nature of a simple miniature wind tunnel; it provides a controlled air stream for experiments which use matching test equipment.

The bench and associated experiments are intended for use in conjunction with lecture courses in fluid mechanics or aerodynamics, particularly in the early stages. The experiments have generally been devised so that they may be set up for classroom demonstration to illustrate a lecture topic. In many cases a simple qualitative demonstration of an effect may be all that is needed; in other cases the lecturer may take one or two specimen results and use the data as the basis for a worked classroom example. Alternatively the experiments may be built into a formal laboratory programme of prescribed work in the subject. Although such programmes are in general disfavour, the expositions presented in this book are in the style of formal reports, as these provide a convenient means of developing the relevant theory and of describing the test apparatus, its range and its capabilities. Those lecturers who wish to use the equipment for project work should have little difficulty in formulating suitable projects; certain questions and suggestions are included which should be helpful in this regard. It should be added that the air flow bench and its associated equipment is in many ways complementary to the hydraulics bench which has been manufactured for many years by Tecquipment. As the basic concepts of hydraulics and of incompressible air flow are identical, some of the experiments on the air flow bench bear marked similarities to their counterparts on the hydraulics bench, although the emphasis might be somewhat different. For example, the Bernoulli theorem experiment in air flow corresponds to the Venturi meter in the range of hydraulics equipment. On the other hand, an experiment on water flow over a weir has no aerodynamic equivalent, and the experiment on boundary layers is probably more relevant in a first course on aerodynamics than in a first course in hydraulics.

The air flow bench, shown on plate 2.1, comprises a fan which draws air from the atmosphere and delivers it along a pipe to an air box which is above the test area. In the pipe there is a valve of the iris shutter type, which may be used to regulate the discharge from the fan.

There is a rectangular slot in the underside of the air box to which various contraction sections may be fitted. The air accelerates as it flows
from the box along the contracting passage, and any unsteadiness or unevenness of the flow at the entry becomes proportionately reduced as the streaming velocity increases towards the test section, which is fitted at the exit of the contraction. Discharge from the test section is in most cases directed towards the bench top, in which a circular hole is provided to collect the air so that it may be led through a duct to the rear of the bench. Where it is desired to take the exhaust right out of the laboratory (if, for example, considerable use is to be made of smoke traces) the exhaust duct may be extended as necessary and an extractor fan fitted at the downstream end if required.

A multitube manometer, indicated schematically in Fig. 2.2, is provided.

![Multitube Manometer Diagram](image)

**Fig. 2.1. Multitube Manometer**

It may be used in the vertical position for classroom demonstration; care has been taken to make the indications visible at a distance. For increased sensitivity the manometer may be inclined to various indexed positions in which the readings are effectively multiplied by convenient factors. The manometer must be levelled before starting an experiment. This may be done using the two front adjusting feet whilst observing:

(i) A spirit level mounted on the manometer stand.

(ii) The manometer liquid level for a constant scale reading across all tubes under static conditions.

The reservoir for manometer liquid is mounted on a vertical rod so that it may be set to a convenient height. It is recommended that the manometer tubes marked A in Fig. 2.2, at the two sides, and the reservoir connection, be normally left open to atmospheric pressure. Pressures p₁, p₂, p₃ ... in tubes 1, 2, 3, ... are then gauge pressures, i.e. pressures measured relative to an atmospheric datum. (Pressures relative to some other chosen datum may be obtained by connecting the reservoir and one manometer tube to the required datum). Before starting to record readings, it is worth making a quick survey of the range of pressures likely to be encountered in a particular test. By adjusting the height of the reservoir and the inclination of the manometer board as the air speed is carefully brought up to the maximum, it is possible to decide on the best inclination for the required sensitivity, and on an appropriate reservoir height which gives all the required pressure readings within the range of the scale. Having established this approximate range of readings, the air supply is then stopped and the manometer reading under still air conditions is adjusted to a convenient whole division of the scale. This greatly facilitates the subsequent observation and recording of pressures, since the subtraction of the scale reading which corresponds to the atmospheric datum then becomes a matter of simple mental arithmetic. The accuracy of the manometer is adequate for all experiments described in subsequent chapters, but if a user wishes to use a micromanometer such as a Betz gauge, there is room on the working surface, and a mains socket is provided.

The bench is mounted on wheels with jacking screws so that it may be moved without difficulty. It requires an earthed, a.c., single phase electrical supply.
4. DRAG MEASUREMENT ON CYLINDRICAL BODIES

Introduction

The resistance of a body as it moves through a fluid is of obvious technical importance in hydrodynamics and aerodynamics. In this experiment we place a circular cylinder in an airstream and measure its resistance, or drag, by three methods. We start by introducing the ideas which underlie these methods.

Consider the cylindrical body shown in cross-section in Fig. 4.1. The reader may be unfamiliar with the idea of a non-circular cylinder. In the present context the word "cylinder" is used to describe a body which is generated by a straight line moving round a plane closed curve, its direction being always normal to the plane of the curve. For example, a pencil of hexagonal cross-section is by this definition a cylinder. The curve shown in Fig. 4.1 represents a section of an oval cylinder. An essential property of a cylinder is that its geometry is two-dimensional: each cross-section is exactly the same as every other cross-section, so that its shape may be described without reference to the dimension along the cylinder axis. We shall use the term circular cylinder to denote the particular and important case of the cylinder of circular cross-section. Motion of the cylinder through stationary fluid produces actions on its surface which give rise to a resultant force. It is usually convenient to analyse these actions from the point of view of an observer moving with the cylinder, to whom the fluid appears to be approaching as a uniform stream. At any chosen point A of the surface of the cylinder, the effect of the fluid may conveniently be resolved into two components, pressure p normal to the surface and shear stress τ along the surface. It is convenient to refer absolute
Let \( U \) denote the uniform speed of the motion and \( \rho \) the density of the fluid, then the dynamic pressure in the undisturbed stream, \( \frac{1}{2} \rho U^2 \), is

\[
\frac{1}{2} \rho U^2 = p_o - p_o
\]

where \( p_o \) is the total pressure in the oncoming stream. This pressure is a useful quantity by which the gauge pressure \( p \) and shear stress \( \tau \) may be non-dimensionalised, and the following dimensionless terms are defined:

- Pressure coefficient \( c_p = \frac{p}{\frac{1}{2} \rho U^2} \) (4.2)
- Skin friction coefficient \( c_f = \frac{\tau}{\frac{1}{2} \rho U^2} \) (4.3)

The combined effect of pressure and shear stress (sometimes called skin friction) gives rise to a resultant force on the cylinder. This resultant may conveniently be resolved into the following components acting at any chosen origin \( C \) of the section as shown on Fig. 4.1:

- A component in the direction of \( U \), called the drag force, of intensity \( D \) per unit length of cylinder.
- A component normal to the direction of \( U \), called the lift force, of intensity \( L \) per unit length of cylinder.
- A moment about the origin \( C \), called the pitching moment, of intensity \( M \) per unit length of cylinder.

These components may be expressed in dimensionless terms by definition of drag, lift, and pitching moment coefficients as follows:

- Drag coefficient \( c_D = \frac{D}{\frac{1}{2} \rho U^2 d} \) (4.4)
- Lift coefficient \( c_L = \frac{L}{\frac{1}{2} \rho U^2 d} \) (4.5)
- Pitching moment coefficient \( c_M = \frac{M}{\frac{1}{2} \rho U^2 d^2} \) (4.6)

in which \( d \) denotes a suitable dimension which characterises the size of the cylinder. In Fig. 4.1 this is shown as the width measured across the cylinder.

normal to \( U \), which is the usual convention. (An important exception is the aerofoil, where the length in the direction of flow or “chord” of the section is used instead). The coefficients \( c_D, c_L \) and \( c_M \) are of prime importance, since they are invariably used for correlating aerodynamic force measurements.

We may see how pressure and skin friction coefficients are related to lift and drag coefficients. Consider an element of length \( \delta s \) of the surface, at a point where the normal is inclined at angle \( \theta \) to the direction of \( U \), as shown on Fig. 4.1. The element of drag \( \delta D \) per unit cylinder length due to \( p \) and \( \tau \) is

\[
\delta D = \delta s (p \cos \theta + \tau \sin \theta)
\]

and integrating this round the whole perimeter yields

\[
D = \int_0^{2\pi} (p \cos \theta + \tau \sin \theta) \, ds
\]

This may now be cast in dimensionless form:

\[
\frac{D}{\frac{1}{2} \rho U^2 d} = \int_0^{2\pi} \left( \frac{p}{\frac{1}{2} \rho U^2} \cos \theta + \frac{\tau}{\frac{1}{2} \rho U^2} \sin \theta \right) \, ds
\]

or

\[
c_D = \int_0^{2\pi} (c_p \cos \theta + c_f \sin \theta) \, ds
\]

Similarly

\[
c_L = \int_0^{2\pi} (-c_p \sin \theta + c_f \cos \theta) \, ds
\]

These results show that the drag of a cylinder may be found by measuring \( p \) and \( \tau \) over the surface and calculating the drag coefficient by equation (4.7). Now it is easy to measure the distribution of \( p \) over a cylinder merely by drilling fine holes into its surface, but measurement of \( \tau \) is a much more difficult task. For the case of the circular cylinder, however, the contribution to drag from shear stress (the ‘skin friction drag’) is found to be very much smaller than from pressure (the ‘pressure drag’) and may safely be neglected. Making this assumption and writing

\[
\delta s = R \delta \theta = \frac{d}{2} \delta \theta
\]

for the circular cylinder of Fig. 4.2 simplifies equation (4.7) to

\[
C_D = \int_0^{2\pi} c_p \cos \theta \left( \frac{d}{2} \right) \, d\theta
\]

\[
C_D = \frac{1}{2} \int_0^{2\pi} c_p \cos \theta \, d\theta
\]
From consideration of symmetry, we may write
\[ C_L = C_M = 0 \quad (4.11) \]
for the circular cylinder. Equation (4.10) allows us to calculate \( C_D \) from the measured pressure distribution over the cylinder surface.

Fig. 4.2. The Circular Cylinder

At the point marked S on Fig. 4.2, the oncoming air stream is brought to rest. S is called the stagnation point, and the streamline arriving at S is the dividing streamline. Moving round the cylinder from S, we expect the velocity over the surface to increase from zero at S, and so, according to Bernoulli's equation, we might expect the pressure and therefore the pressure coefficient to fall. By an analysis which is beyond our scope, the velocity \( u \) over the surface is given in terms of \( \theta \) by the simple equation
\[ \frac{U}{U} = 2 \sin \theta \quad (4.12) \]
provided that the fluid is incompressible and non-viscous.

Writing \( p_a \) as the absolute static pressure at A, Bernoulli's equation is
\[ p_o = p_o + \frac{1}{2} \rho U^2 = p_a + \frac{1}{2} \rho u^2 \quad (4.13) \]

The gauge pressure \( p \) at A is thus
\[ p = p_a - p_o = \frac{1}{2} \rho U^2 - \frac{1}{2} \rho u^2 \]
From equation (4.13)
\[ p = \frac{1}{2} \rho U^2 (1 - 4 \sin^2 \theta) \]
so the pressure coefficient \( c_p \) is
\[ c_p = \frac{\frac{D}{2} \rho U^2}{\frac{1}{2} \rho U^2} = 1 - 4 \sin^2 \theta \quad (4.14) \]
This is the theoretical result for an incompressible, inviscid fluid, and forms the basis of comparison with experimental results.

There is an entirely different way of finding the drag on a cylinder, which depends on the application of the momentum equation to the air flow. This equation is derived in textbooks on the subject, but again for completeness a brief exposition is given here. Consider the flow of a fluid along a duct of width \( 2h \) past a cylindrical body which spans the duct, so that the motion is two-dimensional as indicated in Fig. 4.3. The velocity is \( U \) and the pressure is \( p \) in the oncoming flow. Downstream of the cylinder the velocity is no longer uniform; let the velocity be \( u \) at distance \( y \) from the duct centre line. The pressure across the downstream section is assumed to be uniform and has the value \( p_o \). It is convenient to refer to the space bounded by the upstream section, downstream section and duct walls as the control volume and the surface formed by these boundaries as the control surface.

Fig. 4.3. Application of the Momentum Equation to Flow along a Duct past a Cylindrical Body
The forces in the x-direction acting on the fluid in the control volume are, per unit length of cylinder:

- at the upstream section: \( 2h p_0 \)
- at the downstream section: \( -2h p_e \)
- at the cylinder: \( -D \)

Note the minus sign for the force exerted by the cylinder on the fluid, which is equal and opposite to the force exerted by the fluid on the cylinder. Forces due to shear stress on the walls of the duct and due to the fluid weight are neglected. The momentum flux per unit width over the downstream section is

\[
\int_{-h}^{h} \rho u^2 \, dy
\]

and over the upstream section is

\[
\int_{-h}^{h} \rho U^2 \, dy
\]

Equating the net force in the x-direction to the momentum flux out of the control volume

\[
2h p_0 - 2h p_e - D = \int_{-h}^{h} \rho u^2 \, dy - \int_{-h}^{h} \rho U^2 \, dy
\]

(4.15)

Rearranging and making non-dimensional gives the result

\[
C_D = \frac{D}{\frac{1}{2} \rho U^2 d} = \frac{2h p_0 - p_e}{\frac{1}{2} \rho U^2} + \frac{2}{d} \int_{-h}^{h} \left(1 - \frac{U^2}{U^2}\right) \, dy
\]

(4.16)

The integral may also be made non-dimensional by the substitution

\[
y = \frac{\eta}{h}
\]

(4.17)

so that

\[
\int_{-h}^{h} \left(1 - \frac{U^2}{U^2}\right) \, dy = h \int_{-1}^{1} \left(1 - \frac{\eta^2}{h^2}\right) \, d\eta
\]

and the final result is

\[
C_D = \frac{2h p_0 - p_e}{\frac{1}{2} \rho U^2} + \frac{2}{d} \int_{-1}^{1} \left(1 - \frac{\eta^2}{h^2}\right) \, d\eta
\]

(4.18)

Equation (4.18) provides a means to calculate \( C_D \) from the pressure drop along the duct and the velocity distribution in the wake. Note that the derivation does not restrict the result to pressure drag only; the contributions of both pressure and skin friction forces are contained in the force \( D \) which comes into the momentum equation. The skin friction drag on the walls also contributes to the momentum change and is therefore included in \( D \). It is also worth mentioning that equation (4.18) applies only to the case of flow along a duct where the flow is confined between parallel walls.

The foregoing analysis shows how drag force may be found from pressure distribution over the surface of the cylinder and by measurements in the wake. The results obtained from both of these methods may be compared with the drag measured by direct weighing, and this is described in the next section.

Description of Apparatus and Test Procedure

![Diagram of Apparatus](image)

Fig. 4.4. Diagram of Apparatus

Fig. 4.4 shows the essential components of the working section in which the drag of various bodies may be studied. The body under test is mounted on an arm which extends through a hole in one wall of the working section and which is supported on a flexible link so as to form a balance. The zero of the balance may be adjusted to suit the weight of the particular body under test. There is a simple clamping device to lock the balance arm against a stop when
not in use. At the exit from the working section a Pitot tube may be traversed across the section in a plane normal to the axis of the body. The bodies provided are:

- circular cylinder
- flat plate
- aerofoil section.

The circular cylinder is provided with a fine pressure tapping at one point of its surface. A protractor may be attached to the cylinder and the pressure tapping connected to the manometer, so that by rotating the cylinder about its axis to successive angular positions, the complete pressure distribution round the whole surface may be recorded.

The equipment is set up with the circular cylinder affixed to the balance and the zero adjusted. The balance is then clamped and the wind speed brought up to the maximum. Weights are added to the scale pan of the balance to measure the drag force. It is recommended that exact balance is found by suitably trimming the wind speed rather than by making slight adjustment to the weights in the scale pan. In this way, readings may be obtained at convenient points over the whole range of speeds, as can be seen in Table 4.1. There is inevitably some unsteadiness in the readings, and the clamp should be opened cautiously. As the correct setting is approached, the clamp may be released further, allowing the unrestrained reading to be observed. At each wind speed the total pressure $P_0$ and the static pressure $P_R$ at inlet are recorded. Other bodies may now be mounted on the balance in turn to establish their drags.

The circular cylinder is then mounted with the protractor in place, and the wind speed set at some convenient value near the maximum. The surface pressure $p$ is measured at various settings of the protractor; it is recommended that $5^\circ$ intervals be used for readings over the front half and $10^\circ$ intervals for readings over the rear. The total pressure $P_0$ and static pressure $p_0$ at inlet are observed from time to time and the wind speed readjusted if necessary so that they are kept sensibly constant throughout.

Finally, the wake traverse is made. Again keeping constant inlet conditions, the Pitot tube is traversed across the section, using increments of 2 mm near the centre, increasing to 5 or 10 mm in regions where the total pressure is seen to be substantially constant. The readings in the wake are extremely unsteady because of the high level of turbulence. Some users may wish to damp the oscillations by placing a tubing clip on the flexible connection to the manometer. If this practice is used, care must be taken not to use excessive damping which can give rise to error. It is generally preferable to use too little than too much.

<table>
<thead>
<tr>
<th>Drag force (gf/m)</th>
<th>$\frac{P_0}{N/m^2}$</th>
<th>$\frac{P_R}{N/m^2}$</th>
<th>$\frac{P_0-P_R}{% N/m^2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>655</td>
<td>125</td>
<td>530</td>
</tr>
<tr>
<td>28</td>
<td>585</td>
<td>115</td>
<td>470</td>
</tr>
<tr>
<td>24</td>
<td>500</td>
<td>100</td>
<td>400</td>
</tr>
<tr>
<td>10</td>
<td>395</td>
<td>80</td>
<td>315</td>
</tr>
<tr>
<td>14</td>
<td>300</td>
<td>65</td>
<td>235</td>
</tr>
<tr>
<td>10</td>
<td>210</td>
<td>45</td>
<td>165</td>
</tr>
<tr>
<td>5</td>
<td>100</td>
<td>20</td>
<td>80</td>
</tr>
</tbody>
</table>

Table 4.1 Drag force measured by Direct Weighing.

Air temperature $18^\circ C = 291 K$

Barometric pressure $1016 \text{ mb} = 1.016 \times 10^5 \text{ N/m}^2$

Air density $\rho = \frac{P}{RT} = \frac{1.016 \times 10^5}{287.2 \times 291} = 1.216 \text{ kg/m}^3$

Diameter of cylinder $d = 12.5 \text{ mm} = 0.0125 \text{ m}$

Length of cylinder $l = 48 \text{ mm} = 0.048 \text{ m}$

Half width of working section $\frac{h}{d} = 4$

Table 4.1 gives the drag force measured in units of gram-force by direct weighing at various air speeds. The drag force is written as the product of the drag $D$ per unit length and the length $l$ of the cylinder.

![Fig. 4.5. Measured Drag Force on a Circular Cylinder](image-url)
In Fig. 4.5 the force is plotted against dynamic pressure and a good linear relationship is established with a slope

$$\frac{D}{\frac{1}{2} \rho U^2} = \frac{24.2}{400}$$

$$= \frac{24.2 \times 9.81 \times 10^{-3}}{400} \text{ m}^2$$

$$= 5.94 \times 10^{-4} \text{ m}^2$$

(Note that 1 gmf = 981 dyn = 9.81 \times 10^{-3} N).

$C_D$ may now be found by substitution in equation (4.4)

$$C_D = \frac{D}{\frac{1}{2} \rho U^2 d} = \frac{\frac{D}{\frac{1}{2} \rho U^2 d}}{\frac{1}{2} \rho U^2 dl}$$

$$= \frac{5.94 \times 10^{-4}}{0.0125 \times 0.048} = 0.99$$

Value of drag coefficient by direct weighing $C_D = 0.99$

<table>
<thead>
<tr>
<th>$\theta$ (degree)</th>
<th>$P - P_0$ (N/m)</th>
<th>$P/\frac{1}{2} \rho U^2$</th>
<th>$C_D \cos \theta$</th>
<th>$\theta$ (degree)</th>
<th>$P - P_0$ (N/m)</th>
<th>$P/\frac{1}{2} \rho U^2$</th>
<th>$C_D \cos \theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>445</td>
<td>0.98</td>
<td>0.98</td>
<td>0</td>
<td>445</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
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<td>-5</td>
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<td>0.97</td>
<td>0.97</td>
</tr>
<tr>
<td>10</td>
<td>430</td>
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<td>0.98</td>
<td>-10</td>
<td>430</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td>15</td>
<td>345</td>
<td>0.76</td>
<td>0.59</td>
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<td>370</td>
<td>0.62</td>
<td>0.79</td>
</tr>
<tr>
<td>20</td>
<td>295</td>
<td>0.65</td>
<td>0.51</td>
<td>-29</td>
<td>315</td>
<td>0.66</td>
<td>0.73</td>
</tr>
<tr>
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<td>0.45</td>
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<td>0.45</td>
</tr>
<tr>
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<td>-30</td>
<td>225</td>
<td>0.51</td>
<td>0.39</td>
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<tr>
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<td>133</td>
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<td>40</td>
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<td>0.14</td>
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<td>0.19</td>
<td>0.14</td>
</tr>
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<td>50</td>
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<td>0.13</td>
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<td>0.09</td>
</tr>
<tr>
<td>60</td>
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<td>5</td>
<td>26</td>
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<td>0.08</td>
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<td>0.08</td>
</tr>
<tr>
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<td>0.07</td>
<td>15</td>
<td>18</td>
<td>0.11</td>
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<tr>
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<td>0.10</td>
<td>0.07</td>
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<td>0.06</td>
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<td>0.06</td>
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<td>0.36</td>
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<td>0.08</td>
<td>0.06</td>
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<td>0.06</td>
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<td>6</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>100</td>
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<td>0.05</td>
</tr>
<tr>
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<td>0.05</td>
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<td>0.05</td>
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<td>0.05</td>
<td>0.05</td>
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<td>0.05</td>
<td>65</td>
<td>0</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
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<td>0.05</td>
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<tr>
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<td>0.18</td>
<td>0.05</td>
<td>75</td>
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<td>0.05</td>
<td>0.05</td>
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<tr>
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<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>180</td>
<td>0</td>
<td>0.14</td>
<td>0.05</td>
<td>85</td>
<td>0</td>
<td>0.05</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table 4.2 Pressure Distribution Round Cylinder

Fig. 4.6. Pressure Distribution Round Circular Cylinder

Fig. 4.7. Distribution of $c_D \cos \theta$ Round Cylinder

The measured pressure distribution round the cylinder is given in Table 4.2, and graphs of $c_D$ and $c_D \cos \theta$ as functions of angle $\theta$ from the front are presented on Figs. 4.6 and 4.7. The velocity pressure at inlet during this test was:

$$P_0 - P_0 = 450, 455, 455 : \text{Mean } 453 \text{ N/m}^2$$

$$\therefore \frac{1}{2} \rho U^2 = 453 \text{ N/m}^2$$
The pressure is seen to be fairly symmetrical about the line θ = 0. The pressure coefficient falls over the front portion towards a minimum at θ = 70°, and thereafter rises a little. Over the rear half of the cylinder the pressure is fairly uniform. The distribution of $C_p$ for a cylinder in inviscid, incompressible fluid is shown for comparison, calculated from equation (4.14).

$C_D$ may be obtained from Fig. 4.7 by use of a planimeter to measure the area beneath the curve. It is found to be

$$\int_0^{2\pi} C_p \cos \theta \, d\theta = 2.02$$

(Note that in circular measure the value of θ runs from zero to $2\pi$, so the area enclosed by the rectangle formed by $\theta = 0$, $\theta = \pm 180^\circ$, $C_D = 0$, $C_D = 1$ is $2\pi$).

From equation (4.10)

$$C_D = \frac{1}{\int_0^{2\pi} C_p \cos \theta \, d\theta} = 1.01$$

Value of drag coefficient obtained by pressure plotting = 1.01

<table>
<thead>
<tr>
<th>$\gamma$ (mm)</th>
<th>$\gamma$ h</th>
<th>$p_e - p_o = \frac{\gamma}{\gamma} \rho u^2$ (N/m²)</th>
<th>$u / U$</th>
<th>$u^2 / U^2$</th>
<th>$p_e - p_o = \gamma \rho u^2$ (N/m²)</th>
<th>$u / U$</th>
<th>$u^2 / U^2$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.00</td>
<td>70 0.36 0.87 0.00 70 0.36 0.87</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.02</td>
<td>75 0.37 0.86 -0.02 80 0.39 0.85</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.04</td>
<td>80 0.47 0.78 -0.06 120 0.47 0.78</td>
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<td></td>
<td></td>
<td></td>
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</tr>
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<tr>
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<tr>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
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<td>625 1.08 -0.17 -0.34 630 1.09 0.18</td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>19</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>0.50</td>
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</tr>
<tr>
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<td>645 1.10 -0.21 -0.70 640 1.09 0.20</td>
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<tr>
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<td>365 0.83 0.32 -1.00 420 0.79 0.22</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Table 4.3 Results of Velocity Traverse in Wake

The results of the wake traverse are shown in Table 4.3, taken with constant values of $p_o$ and $p_e$. The static pressure $p_e$ in the plane of the traverse is assumed to be atmospheric.

$$p_o - p_e = 540, 535, 530, 535 \quad \text{Mean} = 535 \text{ N/m}^2$$

$$\frac{1}{2} \gamma \rho U^2 = 535 \text{ N/m}^2$$

$$p_o - p_e = 115, 115, 110, 110 \quad \text{Mean} = 113 \text{ N/m}^2$$

Readings are recorded at successive values of the distance $y$ from the centerline, made dimensionless by dividing by $h$ in the next column. The third column gives the Pitot reading referred to the static pressure $p_e$, and so represents the local velocity pressure $\frac{1}{2} \gamma \rho U^2$ at a point in the exit section. This is also made dimensionless by dividing by $\frac{1}{2} \gamma \rho U^2$, the velocity pressure at inlet, and the next two columns show $\frac{U}{U}$ and $1 - \frac{u^2}{U^2}$. In the first line of the table, for example,

$$\frac{U}{U} = \frac{70}{635} = 0.13$$

$$1 - \frac{u^2}{U^2} = 0.36$$

Fig. 4.8 shows the results in graphical form. Note that $\frac{U}{U}$ falls to about 0.4 on the center line. Outside the wake, $\frac{U}{U}$ has a constant value somewhat in excess of 1.0, with some evidence of boundary layers on the walls. By using a planimeter, the area beneath the curve is found to be

$$\int_1^h \frac{U}{U} \, d\eta = 1.974$$

The expected value of this integral is 2.0, since the equation of continuity requires the same volume flow rate over the outlet section as over the inlet section, viz.

$$\int_0^h \frac{U}{U} \, dy = 2U \, h$$

$$\int_1^h \frac{U}{U} \, d\eta = 2$$
The measured value implies a mean velocity \( \bar{u} \) at the exit section given by

\[
\frac{\bar{u}}{U} = \frac{1.974}{2} = 0.987, \text{ say 0.99}
\]

The equation of continuity requires that \( \frac{\bar{u}}{U} = 1 \), so the wake measurements satisfy continuity with an error of about 1%.

The drag coefficient may be obtained by use of the curve of \( 1 - \frac{u^2}{U^2} \) in Fig. 4.8. The area beneath the curve is found by the planimeter to be

\[
\int_{-1}^{1} \left( 1 - \frac{u^2}{U^2} \right) \, d\eta = -0.074
\]

Using equation (4.18) gives

\[
C_D = \frac{2h}{d} \frac{p_D - p_e}{\frac{1}{2} \rho U^2} + \frac{2h}{d} \int_{-1}^{1} \left( 1 - \frac{u^2}{U^2} \right) \, d\eta
\]

\[
= 2 \times 4 \times \frac{113}{535} - 2 \times 4 \times 0.074 = 1.10
\]

Value of \( C_D \) from wake traverse \( C_D = 1.10 \)

Discussion

The values obtained for drag coefficient of the circular cylinder are as follows

by direct weighting \( C_D = 0.99 \)
by pressure plotting \( C_D = 1.01 \)
by wake traverse \( C_D = 1.10 \)

Taking the first of these as being the most reliable, we see that pressure plotting yields a result within the probable limits of accuracy. The wake traverse is about 10% different from the other two. Although the readings of total pressure are extremely unsteady in the wake, the resulting curves of Fig. 4.8 are quite smooth, and the equation of continuity is reasonably well satisfied by the velocity distribution. This leads to the conclusion that the error is most likely to be in the term \( (p_D - p_e)/\frac{1}{2} \rho U^2 \) on which the calculation of \( C_D \) also depends. Possibly the planes of measurement are not sufficiently far from the cylinder for the assumption of uniform pressure over the inlet and the outlet sections to be valid. The skin friction drag on the walls, neglected in the derivation of equation (4.18), also has the effect of increasing the apparent value of \( C_D \).

There have been very many measurements of drag of circular cylinders, and the variation of \( C_D \) with Reynolds number \( Re \), where

\[
Re = \frac{Ud}{\nu}
\]

is well established. In this experiment, the absolute viscosity is, at 18°C,

\[
\mu = 1.80 \times 10^{-5} \text{ kg/m s}
\]

so the kinematic viscosity is

\[
\nu = \frac{\mu}{\rho} = \frac{1.80 \times 10^{-5}}{1.216} = 1.48 \times 10^{-5} \text{ m}^2/\text{s}
\]

The velocity \( U \) at the typical value of velocity pressure

\[
\frac{1}{2} \rho U^2 = 500 \text{ N/m}^2
\]

is \( U = \left( \frac{2 \times 500}{1.216} \right)^{\frac{1}{2}} = 28.7 \text{ m/s} \)

So a typical value of \( Re \) for these test is
Re \[= \frac{U_d}{\nu} = \frac{28.7 \times 0.0125}{1.48} \times 10^6\]

Re \[= 2.4 \times 10^4\]

It is well established that in the range of Re from \(10^4\) to \(10^6\), \(C_D\) is almost constant, the value usually quoted for a cylinder being

\[C_D = 1.20\]

This is for the case of a long cylinder, for which \(\frac{L}{d}\) is so great that the three-dimensional flows at the ends have no significant effect on the result. Our experiments are made for a cylinder with \(\frac{L}{d} = 3.9\) for which end effects must be significant. Moreover, the cylinder presents an appreciable blockage to the cross section of the flow — its projected area is about 1/8th of the cross-sectional area of the working section, so the results cannot be compared directly with those obtained on cylinders in unconfined flows.

Questions for Further Discussion

1. A yawmeter is an instrument for finding the direction of a fluid stream. One type of yawmeter consists of a circular cylinder with two surface holes at different positions in the same diametral plane. It is placed in the stream and rotated slowly about its axis until a balance is obtained between the observed pressures at the holes. Suggest a suitable angular spacing between the holes to give good sensitivity.

2. Measure the drag coefficients of a flat plate and an aerofoil section by direct weighing, and comment on the results obtained. Could pressure plotting be used to establish drag coefficients of these sections? (\(C_D = 2.0\) for plate, \(C_D = 0.04\) for aerofoil)

3. The circular cylinder presents a substantial blockage to the flow along the working section. Suppose instead of using the approach velocity \(U\), the velocity \(U_1\) past the cylinder were used as the velocity on which the reference value of velocity head is based. Show that

\[U_1 = \frac{8}{7} U\]

for results presented here, and find the corresponding value of \(C_{D_1}\) from

\[C_{D_1} = \frac{D}{\frac{2}{\pi} U_1^2 d} \quad (0.76)\]
6. BOUNDARY LAYERS

Introduction

It is a fact well-established by experiment that when a fluid flows over a solid surface there is no slip at the surface. The fluid in immediate contact with a surface moves with it, and the relative velocity increases from zero at the surface to the velocity in the free stream through a layer of fluid which is called the boundary layer.

Consider steady flow over a flat smooth plate as shown in Fig. 6.1, where the streaming velocity $U$ is constant over the length of the plate. It is found that the thickness of the boundary layer grows along the length of the plate as indicated on the figure. The motion in the boundary layer is laminar at the start, but if the plate is sufficiently long, a transition to turbulence is observed. This transition is produced by small disturbances which, beyond a certain distance, grow rapidly and merge to produce the apparently random fluctuations of velocity which are characteristics of turbulent motions. The parameter which characterises the position of the transition is the Reynolds number $Re_x$ based on distance $x$ from the leading edge:

$$Re_x = \frac{UX}{v}$$  \hspace{1cm} (6.1)

Plate 6.1. Boundary Layer Apparatus.

Fig. 6.1. General Characteristics of Boundary Layer over a Flat Plate.
The nature of the process of transition renders it prone to factors such as turbulence in the free stream and surface roughness of the boundary, so it is not possible to give a single value of $Re_v$ at which transition will occur, but it is usually found in the range $1 \times 10^5$ to $5 \times 10^5$.

Definitions of thickness

A little consideration will show that the boundary layer thickness $\delta$, shown in fig. 6.1 as the thickness where the velocity reaches the free stream value, is not an entirely satisfactory concept. The velocity in the boundary layer increases towards $U$ in an asymptotic manner, so the distance $y$ at which we might consider the velocity to have reached $U$ will depend on the accuracy of measurement. A much more useful concept of thickness is the so-called displacement thickness $\delta^*$. This is defined as the thickness by which fluid outside the layer is displaced away from the boundary by the existence of the layer, as indicated schematically in fig. 6.2, by the streamline approach.

![Fig. 6.2. Velocity Distribution and Displacement Thickness of Boundary Layer](image)

In fig. 6.2 the distribution of velocity $u$ within the layer is shown as a function of distance $y$ from the boundary as curve OA. If there were no boundary layer, the free stream velocity $U$ would persist right down to the boundary as shown by the line CA. The reduction in volume flow rate (per unit width normal to the diagram) due to the reduction of velocity in the layer is therefore

$$\Delta Q = \int_0^h (U - u) \, dy \quad (6.2)$$

which is the shaded area OAC in the figure, the dimension $h$ being chosen so that $u = U$ for any value of $y$ greater than $h$. If the volume flow rate is now considered to be restored by displacement of the streamline at A'A away from the surface to a position B'B through a distance $\delta^*$, the volume flow rate between A'A and B'B is also $\Delta Q$, and this is seen to be

$$\Delta Q = U \delta^* \quad (6.3)$$

Equating the results of equation (6.2) and (6.3) gives

$$\delta^* = \frac{1}{U} \int_0^h (U - u) \, dy$$
or

$$\delta^* = \int_0^h (1 - \frac{u}{U}) \, dy$$

Now $h$ is any arbitrary value which satisfies the condition

$$u = U$$
only

$$1 - \frac{u}{U} = 0$$

for all values of $y$ greater than $h$. The value of $h$ may therefore be increased indefinitely without affecting the value of the integral, so we allow $h$ to increase towards infinity, viz:

$$h \to \infty$$

and obtain the result

$$\delta^* = \int_0^\infty (1 - \frac{u}{U}) \, dy$$

(6.4)

We shall see that in the practical measurement of $\delta^*$ from a measured velocity distribution the infinite upper limit presents no difficulty.

A further definition is required when momentum effects within the boundary layer are considered. Consider a control volume of length $\delta x$, height $h$ (greater than the boundary layer thickness $\delta$) and unit thickness normal to the plane of the diagrams shown in fig. 6.3. The rate of mass inflow is $\dot{m}$ at the left-hand end, and the rate of mass outflow at the right-hand end is $\frac{d\dot{m}}{dx} \delta x$. Consideration of continuity then shows the outflow through the upper surface to be $-\frac{d\dot{m}}{dx} \delta x$. The momentum equation may now be written.

![Fig. 6.3. Mass and Momentum Flux in Boundary Layer](image)
The net rate of efflux of x-component of momentum $\dot{M}$ from the control volume is the sum of

\[ \dot{M} + \frac{d\dot{M}}{dx} \delta x \] at the right-hand end

\[ - \dot{M} \] at the left-hand end

and \(-U \frac{d\dot{M}}{dx} \delta x\) at the upper surface (noting that the x-component of velocity is $U$, and using the mass outflow rate previously found).

If the surface shear stress is $\tau_w$ acting in the direction shown in the diagram, the momentum equation is then

\[ -\tau_w \delta x = \dot{M} + \frac{d\dot{M}}{dx} \delta x - \dot{M} - U \frac{d\dot{M}}{dx} \delta x \]

which simplifies to

\[ \tau_w = U \frac{d\dot{M}}{dx} - \frac{d\dot{M}}{dx} \delta x \]

or

\[ \tau_w = \frac{d}{dx} (U\dot{M} - \dot{M}) \] (6.5)

Now $\dot{M} = \rho \int_o^h udy$ \hfill (6.6)

and $\dot{M} = \rho \int_o^h u^2 dy$ \hfill (6.7)

so substituting these results into equation (6.5) gives

\[ \tau_w = \rho \frac{d}{dx} \left[ \int_o^h (Uu - u^2) dy \right] \]

or

\[ \tau_w = \rho U^2 \frac{d}{dx} \left[ \int_o^h \frac{u}{U} \left(1 - \frac{U}{u} \right) dy \right] \]

Since $u = U$ for all values of $y$ greater than $h$, the arbitrary upper limit may be replaced by infinity, giving

\[ \tau_w = \rho U^2 \frac{d}{dx} \left[ \int_o^\infty \frac{u}{U} \left(1 - \frac{U}{u} \right) dy \right] \] (6.8)

It is convenient to express $\tau_w$ in the dimensionless form of a local skin friction coefficient

\[ c_f = \frac{\tau_w}{\frac{1}{2} \rho U^2} \] \hfill (6.9)

and if this is done, equation (6.8) becomes

\[ c_f = 2 \frac{d}{dx} \left[ \int_o^\infty \frac{u}{U} \left(1 - \frac{U}{u} \right) dy \right] \] \hfill (6.10)

i.e. writing of this result is simplified if we now define

\[ \Theta = \int_o^\infty \frac{u}{U} \left(1 - \frac{U}{u} \right) dy \] \hfill (6.11)

where $\Theta$ is known as the momentum thickness of the boundary layer, and equation (6.10) becomes

\[ c_f = 2 \frac{d\Theta}{dx} \] \hfill (6.12)

The total skin friction force per unit width on a plate of length $L$ is

\[ D_f = \int_o^L \tau_w dx \] \hfill (6.13)

Writing $\tau_w$ in terms of $c_f$ from equation (6.9)

\[ D_f = \frac{1}{2} \rho U^2 \int_o^L c_f dx \]

and from equation (6.12),

\[ D_f = \frac{1}{2} \rho U^2 \times 2 \int_o^L \frac{d\Theta}{dx} dx \]

When $x = 0$, $\Theta = \Theta_L$, and writing $\Theta_L$ for the momentum thickness at distance $L$ from the leading edge,

\[ D_f = \frac{1}{2} \rho U^2 \times 2 \Theta_L \] \hfill (6.14)

The skin friction force $D_f$ is now written in terms of a dimensionless overall skin friction coefficient $C_F$ where

\[ C_F = \frac{D_f}{\frac{1}{2} \rho U^2 L} \]
and substituting $D_f$ from equation (6.14) gives

$$C_f = \frac{2\Theta L}{L} \quad (6.15)$$

This equation gives the overall skin friction coefficient on a flat plate very simply in terms of the momentum thickness at the trailing edge and the length of the plate.

It is frequently useful to refer to the ratio of displacement thickness $\delta^*$ to momentum thickness $\Theta$, and this is called the shape factor $H$:

$$H = \frac{\delta^*}{\Theta} \quad (6.16)$$

The calculation of the velocity profiles and thickness of boundary layers is beyond our present scope, but for reference and for comparison with results of experiments a few results are presented here. For a laminar boundary layer along a flat plate with uniform free stream velocity, the velocity profile has been calculated and some numerical results are presented in table 6.1.

<table>
<thead>
<tr>
<th>$y/Re_x$</th>
<th>0</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>3.0</th>
<th>4.0</th>
<th>5.0</th>
<th>6.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u/U$</td>
<td>0.01659</td>
<td>0.33389</td>
<td>0.48689</td>
<td>0.62989</td>
<td>0.69861</td>
<td>0.65565</td>
<td>0.6186</td>
<td>0.59990</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.1 Velocity distribution in laminar boundary layer along flat plate.

Note the dimensionless parameter $y/Re_x/\delta^*$ used in the table, which generalises the results to any value of distance $x$ along the plate. The displacement thickness $\delta^*$ and the momentum thickness $\Theta$ are given by

$$\delta^* = 1.721x/Re_x \quad (6.17)$$

and $\Theta = 0.664x/Re_x \quad (6.18)$

from which it may be noted that the thickness along the plate grows in proportion to $\sqrt{x}$. The shape factor is

$$H = 2.59 \quad (6.19)$$

For a turbulent boundary layer along a smooth flat plate there are no corresponding calculated results. Frequently the velocity distribution is expressed in the form

$$\frac{u}{U} = \left(\frac{x}{\delta^*}\right)^{\frac{1}{n}} \quad (6.20)$$

where $n$ is an index which varies from about 5 to 8 as the value of $Re_x$ increases in the range of $10^3$ to $10^4$, although there are many alternative expressions. The displacement and momentum thicknesses are frequently quoted as

$$\delta^* = 0.046x/(Re_x)^{0.2} \quad (6.21)$$

$$\Theta = 0.036x/(Re_x)^{0.2} \quad (6.22)$$

with the shape factor

$$H = 1.29 \quad (6.23)$$

The Effect of Pressure Gradient

The preceding discussion has related to boundary layer development along a smooth plate with uniform flow in the free stream, i.e. in conditions of zero pressure gradient along the plate. If the free stream is accelerating or decelerating, substantial changes take place in the boundary layer development. For an accelerating free stream, the pressure falls in the direction of flow, the pressure gradient being given by differentiating Bernoulli's equation in the free stream as

$$\frac{dp}{dx} = -\rho U \frac{dU}{dx} \quad (6.24)$$

The boundary layer grows less rapidly than in zero pressure gradient and transition to turbulence is inhibited. For a decelerating free stream, the reverse effects are observed. The boundary layer grows more rapidly and the shape factor increases in the downstream direction. The pressure rise in the direction of flow, and this pressure rise tends to retard the fluid in the boundary layer more severely than that in the main stream since it is moving less quickly. Energy diffuses from the free stream through the outer part of the boundary layer down towards the surface to maintain the forward movement against the rising pressure. However, if the pressure gradient is sufficient to steep, this diffusion is insufficient to sustain the forward movement, and the flow along the surface reverses, forcing the main stream to separate. It is this separation, or stall as it is sometimes called, which leads to the main component of drag on bluff bodies and to the collapse of the lift force on an aerfoil when the angle of incidence is excessive.

Description of Apparatus

Fig. 6.4 shows the arrangement of the test section attached to the outlet of the contraction of the air flow bench. A flat plate is placed at mid height
in the section, with a sharpened edge facing the oncoming flow. One side of the plate is smooth and the other is rough so that by turning the plate over, results may be obtained on both types of surface.

A fine Pitot tube may be traversed through the boundary layer at a section near the downstream edge of the plate. This tube is a delicate instrument which must be handled with extreme care if damage is to be avoided. The end of the tube is flattened so that it presents a narrow slit opening to the flow. A low voltage electrical circuit is provided to indicate when the tube is touching the metal surface of the plate. The traversing mechanism is spring loaded to prevent backlash and a micrometer reading is used to indicate the displacement of the Pitot tube.

Liners may be placed on the walls of the working section so that either a generally accelerating or generally decelerating free stream may be produced along the length of the plate, depending on which way round they are fitted. With the liners removed, uniform free-stream flow conditions obtain over the plate length.

To obtain a boundary layer velocity profile, the Pitot tube is set at about 10 mm distance from the surface and the desired wind speed is established by bringing the pressure \( P_0 \) in the air box to the required value. Readings of total pressure \( P \) measured by the Pitot tube are then recorded over a range of settings of the micrometer as the tube is traversed towards the plate. At first the readings should be substantially constant, indicating that the traverse has been started in the free stream; if this is not the case, go back and start with an initial setting further from the plate. As the Pitot tube reading begins to fall, the step length of the traverse should be reduced so that at least 10 readings are obtained over the range of reducing readings. The reading does not fall to zero as the tube touches the wall because of its finite thickness, so the traverse is stopped as soon as contact is indicated either by the electrical circuit or by the readings becoming constant as the micrometer is advanced towards the surface.

Readings obtained in turbulent boundary layers are subject to unsteadiness which leads to difficulty in obtaining average readings on the manometer. Damping may be provided by squeezing the connecting plastic tube, but care should be taken that the restriction is not too severe, which can lead to false readings.

**Results and Calculations**

(a) Turbulent boundary layers on smooth and rough surfaces

The plate was installed in the test section without the liners fitted, and measurements were made in the boundary layer formed on the smooth surface and then on the rough surface.

- Air temperature \( 19^\circ \text{C} \) = 292 K
- Barometric pressure \( 1010 \text{ mb} \) = \( 1.010 \times 10^5 \text{ N/m}^2 \)
- Air density \( \rho = \frac{1.010 \times 10^5}{287.2 \times 292} \) = 1.204 kg/m\(^3\)
- Coefficient of viscosity \( \mu = 1.80 \times 10^{-5} \text{ kg/m s} \)
- Coefficient of kinematic viscosity \( \nu = \frac{\mu}{\rho} = 1.49 \times 10^{-5} \text{ m}^2/\text{s} \)
- Length of plate from leading edge to traverse section, \( L = 0.265 \text{ m} \)
- Thickness of Pitot tube at tip, \( 2t = 0.40 \text{ mm} \)
- Displacement of tube centre from surface when in contact, \( t = 0.20 \text{ mm} \)
- Pressure in air box: 640, 640, 640, 640 N/m\(^2\)
Readings of Pitot pressure $P$ are tabulated in Tables 6.2 and 6.3. Values of $y$ shown in the tables are obtained from the micrometer reading at which the tube just touched the surface, making allowance for the initial displacement of the Pitot tube. Values of $u/U$ are found from

$$\frac{u}{U} = \sqrt{\frac{P}{P_0}}$$

where $P_0$ is the Pitot tube reading in the free stream.

The free stream velocity $U$ is obtained from:

$$\frac{\gamma \rho U^2}{2} = 550 \text{ N/m}^2$$

$$\therefore U = \sqrt{\frac{2 \times 550}{1.204}} = 30.2 \text{ m/s}$$

$$Re = \frac{UL}{v} = \frac{30.2 \times 0.265 \times 10^5}{1.49} = 5.37 \times 10^5$$

<table>
<thead>
<tr>
<th>Micrometer Reading (mm)</th>
<th>$y$ (mm)</th>
<th>$P$ (N/m²)</th>
<th>$u/U$</th>
</tr>
</thead>
<tbody>
<tr>
<td>21.0</td>
<td>6.06</td>
<td>550</td>
<td>1.00</td>
</tr>
<tr>
<td>20.0</td>
<td>5.06</td>
<td>550</td>
<td>1.00</td>
</tr>
<tr>
<td>19.0</td>
<td>4.06</td>
<td>550</td>
<td>1.00</td>
</tr>
<tr>
<td>18.0</td>
<td>3.08</td>
<td>530</td>
<td>0.98</td>
</tr>
<tr>
<td>17.0</td>
<td>2.08</td>
<td>495</td>
<td>0.85</td>
</tr>
<tr>
<td>16.5</td>
<td>1.56</td>
<td>460</td>
<td>0.81</td>
</tr>
<tr>
<td>16.0</td>
<td>1.06</td>
<td>415</td>
<td>0.87</td>
</tr>
<tr>
<td>15.8</td>
<td>0.85</td>
<td>389</td>
<td>0.83</td>
</tr>
<tr>
<td>15.6</td>
<td>0.66</td>
<td>360</td>
<td>0.81</td>
</tr>
<tr>
<td>15.4</td>
<td>0.46</td>
<td>320</td>
<td>0.76</td>
</tr>
<tr>
<td>15.2</td>
<td>0.26</td>
<td>250</td>
<td>0.67</td>
</tr>
<tr>
<td>15.14</td>
<td>0.20</td>
<td>180</td>
<td>0.57</td>
</tr>
</tbody>
</table>

Table 6.2  Velocity Distribution in Boundary Layer on Smooth Flat Plate, $Re = 5.37 \times 10^5$

<table>
<thead>
<tr>
<th>Micrometer Reading (mm)</th>
<th>$y$ (mm)</th>
<th>$P$ (N/m²)</th>
<th>$u/U$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25.0</td>
<td>9.10</td>
<td>540</td>
<td>1.00</td>
</tr>
<tr>
<td>24.0</td>
<td>8.10</td>
<td>540</td>
<td>1.00</td>
</tr>
<tr>
<td>23.0</td>
<td>7.10</td>
<td>525</td>
<td>0.99</td>
</tr>
<tr>
<td>22.0</td>
<td>6.10</td>
<td>515</td>
<td>0.98</td>
</tr>
<tr>
<td>21.0</td>
<td>5.10</td>
<td>500</td>
<td>0.96</td>
</tr>
<tr>
<td>20.0</td>
<td>4.10</td>
<td>470</td>
<td>0.93</td>
</tr>
<tr>
<td>19.0</td>
<td>3.10</td>
<td>420</td>
<td>0.88</td>
</tr>
<tr>
<td>18.5</td>
<td>2.60</td>
<td>375</td>
<td>0.83</td>
</tr>
<tr>
<td>18.0</td>
<td>2.10</td>
<td>335</td>
<td>0.79</td>
</tr>
<tr>
<td>17.5</td>
<td>1.60</td>
<td>275</td>
<td>0.71</td>
</tr>
<tr>
<td>17.0</td>
<td>1.10</td>
<td>215</td>
<td>0.63</td>
</tr>
<tr>
<td>16.5</td>
<td>0.60</td>
<td>150</td>
<td>0.53</td>
</tr>
<tr>
<td>16.3</td>
<td>0.40</td>
<td>125</td>
<td>0.48</td>
</tr>
<tr>
<td>16.10</td>
<td>0.20</td>
<td>100</td>
<td>0.43</td>
</tr>
</tbody>
</table>

Table 6.3  Velocity Distribution in Boundary Layer on Rough Flat Plate, $Re = 5.37 \times 10^5$

Fig. 6.5 shows the velocity distributions plotted for both smooth and flat plates. Also shown are the curves of $u/U(1 - u/U)$, which are easily deduced from the curves of $u/U$ by reference to the following table:

<table>
<thead>
<tr>
<th>$\frac{u}{U}$</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{u}{U}(1 - u/U)$</td>
<td>0.25</td>
<td>0.24</td>
<td>0.21</td>
<td>0.18</td>
<td>0.09</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 6.4  Values of $u/U(1 - u/U)$
The appropriate areas under the curves measured by planimeter are:

Smooth plate:
\[ \delta^* = \int \left(1 - \frac{u}{U}\right) dy = 0.53 \text{ mm} \]
\[ \Theta = \int \frac{u}{U} \left(1 - \frac{u}{U}\right) dy = 0.40 \text{ mm} \]
\[ \therefore H = \frac{\delta^*}{\Theta} = 1.32 \]

Rough plate:
\[ \delta^* = \int \left(1 - \frac{u}{U}\right) dy = 1.50 \text{ mm} \]
\[ \Theta = \int \frac{u}{U} \left(1 - \frac{u}{U}\right) dy = 0.98 \text{ mm} \]
\[ \therefore H = \frac{\delta^*}{\Theta} = 1.53 \]

On fig. 6.5 a 1/7th power law is shown, corresponding to
\[ \frac{u}{U} = \left( \frac{y}{3} \right)^{\frac{1}{7}} \]
and this is seen to compare reasonably well with the experimental results on the smooth plate. For the rough plate, however, the velocity distribution does not fall towards zero at \( y = 0 \). This is because the origin of the traverse has been taken from the highest points of the rough surface. An examination of the structure of the roughness would be required to establish the position of the mean surface from which \( y \) should be measured.

The values of \( \delta^* \) and \( \Theta \) calculated for a turbulent boundary layer from equations (6.21) and (6.22) are, with the length \( L = 0.265 \text{ m} \) inserted,
\[ \delta^* = 0.046 \times \frac{2}{0.265} \times (5.37 \times 10^5)^{0.2} = 0.87 \times 10^{-3} \text{ m} = 0.87 \text{ mm} \]
\[ \Theta = 0.036 \times \frac{2}{0.265} \times (5.37 \times 10^5)^{0.2} = 0.68 \times 10^{-3} \text{ m} = 0.68 \text{ mm} \]
The experimental results for the smooth plate are noticeably lower than these values, indicating that over part of the length of the surface the boundary layer is laminar, yielding an overall skin friction less than if the whole length of the layer were turbulent.

For the rough plate the boundary layer thickness is more than twice that on the smooth plate. Also it is more than the calculated values, showing that the roughness has produced a significant increase in skin friction drag to a value higher than could be obtained on a smooth plate even if the whole length of the boundary layer were turbulent.

(b) Effect of pressure gradient

The test was repeated with the liners fitted to give a generally decelerating flow over the plate length. The Reynolds number based on the main stream velocity at the exit and the length of the plate was
\[ \text{Re} = 4.90 \times 10^5 \]
which is not sufficiently different from the previous value to affect the results. The procedure is the same as before, so full details of the working are not presented. Fig. 6.6 shows the measured velocity profile in comparison with the previous results, from which it is clear that the layer has grown appreciably thicker in the rising pressure which is produced by the decelerating flow. The thickness and shape factor are:
Suggestions for Further Experiment

1. Investigate the effect of a falling pressure gradient on the boundary layer by repeating the tests with the liners reversed.

2. It has been suggested that the velocity distribution in a turbulent boundary layer may be approximated by a logarithmic profile

\[
\frac{u}{U} = A + B \log \left( \frac{y}{\delta} \right)
\]

where A and B are constants. Check whether your results fit this expression.

3. Observe the growth of boundary layer along a plate with a constant main stream velocity by making traverses at successive stations along the plate length. (The plate may be withdrawn from the working section to various positions to allow traverses to be made at different stations along it.) Plot the growth of \( \Theta \) along the plate and consider what information this gives about the skin friction coefficient \( \epsilon \).

4. Consider the possibility of making measurements of laminar boundary layers in this apparatus. If the minimum main stream velocity for which reasonably accurate velocity traverses may be obtained is 10 m/s, and the laminar layer along the plate persists up to \( \text{Re} = 1 \times 10^9 \), show that the layer will extend about 0.15 m from the leading edge. Show also that the displacement thickness at this section would be about 0.8 mm according to equation 6.17.

Summary and Conclusions

Velocity traverses in turbulent boundary layers have been made with a specially shaped fine Pitot tube traversed close to the surface. The resulting velocity profiles have shown that roughness of the plate surface, and a rising pressure gradient, both serve to increase the rate of growth of the boundary layer.