

PARTICLE IMAGE VELOCIMETRY

<http://www.nd.edu/~dtl/cheg459/pivexperiment/>

Particle Image Velocimetry (PIV) is a noninvasive method for measuring local velocities in a flow field. PIV is used in laboratory environments to measure various flow phenomena under a wide range of conditions. In industrial settings, PIV finds use for measurements in hostile environments where other types of sensors might not survive or in flows for which conventional measurement systems may disrupt the flow field. A key advantage of PIV is that it is capable of simultaneously determining a two-dimensional velocity profile map over a fairly wide area. This makes it particularly valuable for time-dependent flows.

The objective of this experiment is to use a PIV apparatus to examine the flow produced by natural convection from a line source of energy. The derivation of the theoretical velocity profile is attached, and you should make yourself thoroughly familiar with these results prior to the first laboratory session. Using the PIV measurements you are to quantitatively test all the key predictions of the theoretical prediction including velocity profile width, magnitude, and variation with distance from the energy source. This experiment is quite demanding, however mostly in the set-up and analysis phases – the actual collection of images for analysis is extremely fast. Thus, you **must not** wait to the last minute to analyze your data – or you will certainly run out of time!

Experimental Apparatus

A laser is combined with a series of mirrors and lenses to produce a thin sheet of light that illuminates the flow field. A small quantity of tracer particles (chalk works quite well) serve as scattering centers. The motion of these particles is tracked by a digital video camera. A short video clip is saved as a sequence of jpg images onto the drive. These image sequences are analyzed by a particle tracking algorithm written in matlab that is provided to you. The detailed procedure for acquiring the images and analyzing them is provided in an appendix to this document. Calibration images are also required so that you can determine where the center of the image is relative to the wire, and to determine x (up) and y (horizontal) length scales. A calibration program and a ruler are also provided.

The flow field is produced by electrical dissipation in a filament. The filament is very fragile, so extreme care should be exercised when adjusting the filament location. It is never a good idea to try stirring the fluid in the tank with the wire inserted! The best working fluids we have found are mixtures of glycerin and distilled water (about three liters are required) yielding a range of fluid Prandtl numbers and material properties. The particle tracking works fairly well for pure distilled water up to equal parts by volume mixtures. The solutions should be freshly prepared each laboratory session: add the chosen amount of water and glycerin as well as the scattering particles (0.04g of the chalk is about the right amount) to the plastic jar and shake it up (*very* vigorously, as viscous fluids are hard to mix!). Pour the fluid into the measurement container, insert the wire cradle, and connect the wires to the power supply. The velocity profile is a function of the power dissipated in the wire, so be sure to note the voltage and current (and wire length), as you will need these to compare with the theoretical predictions.

Experimental Objectives

The purpose of the experiment is to test the validity of the theoretically calculated velocity profile. While the theory is quite elegant, no theory is perfect. In particular, in order to make the problem tractable, the theoretical solution makes a number of approximations which are of questionable applicability to our experimental system. Your job is to determine what the model gets right and what it gets wrong. In the latter case, you should try to figure out -why- the deviation occurs: is it experimental error? Are there physical realities not accounted for in the assumptions? How could the model be improved? Can the analysis of the data be altered in such a way as it more closely corresponds to the theory and better reveals causes of any discrepancy? Note that these are all terribly open-ended questions, and thus there isn't going to be any one right answer to any of them. The experimental analysis requires a lot of thought, creativity, and insight. Have fun!

Specific points of comparison to think about: The theoretical model quantitatively predicts the velocity profile in the x and y directions as a function of the power dissipated in the wire and the fluid properties. For the vertical velocity, you should look at its quantitative magnitude and quantitative profile width, and the growth in magnitude and width as you move further above the wire. Does it obey the expected power-law relationship? The model neglects any back-flow (e.g., it is for infinite container width). Does this influence the magnitude and width of the profile? Can you come up with alternative ways of analyzing the data which might better account for this effect? For the horizontal velocity (much harder to get accurately!) you can probably most easily look at the slope of the velocity near the centerline of the flow field (regression helps here, and averages out a lot of the noise!). Is the magnitude and x dependence about what was predicted? To look at the effect of power and fluid properties, try changing the power level and see what the effect on vertical velocity profile magnitude and width is. Does it match the expected power law dependence? Boundary layer development is a fascinating phenomenon. The theoretical description is only valid after the boundary layer becomes established (e.g., sufficiently far above the wire). By looking very close to the wire you can see how the boundary layer approximation breaks down. What predictions fail first? What predictions does the theory get right closest to the wire? The effect of fluid properties can be determined by varying the fluid composition: again, does the effect match what is predicted? A list of possible questions you could address in your experiments and analysis are provided on the experiment website.

The order in which you do the experiments and analysis is up to you, however I would recommend that you devote the first period to obtaining a complete map of the velocity field at all heights for a particular fluid and power combination. You should analyze the images, and produce a quiver plot of the composite velocity field. You will need to plot up velocity profiles for different heights above the wire in dimensional form, so that you can see how height above the wire influences magnitude and profile width. You should also render the velocities and horizontal positions dimensionless, scale all these quantities with the appropriate power of x as predicted by the theory, and plot up the scaled vertical and horizontal velocities as a function of z . In an ideal world they would all collapse onto the theoretical relations! In the second period you should focus on the effect of power on the velocity field at one or several heights above the wire, determining how the magnitude and width depend on the power. Again, look at both

dimensional and dimensionless velocity profile plots. In the third period you should explore the effect of fluid composition. Start with a high concentration of glycerin and measure the velocity profile at some height above the wire for which good measurements can be easily obtained. Then successively dilute the solution (outside the tank - it needs to be well-mixed!) until you work with a fairly dilute solution, and see how the fluid properties (mainly viscosity, but also heat capacity and other properties) affect the flow field.

Your final report should address the experimental objectives mentioned above.

Safety Precautions

Laser radiation is extremely intense and can cause permanent eye damage if care is not used. **Never** look directly into the laser beam. To avoid specular reflections, do not wear wristwatches or other hand jewelry when manipulating objects near the laser beam. Above all, use common sense.

Apart from the laser, the primary safety hazard is the glycerin. Glycerin is non-toxic (it is often used in food), but it is *very* slippery. Inevitably a few drops will spill to the floor, and such drops need to be cleaned up immediately!

Literature Sources

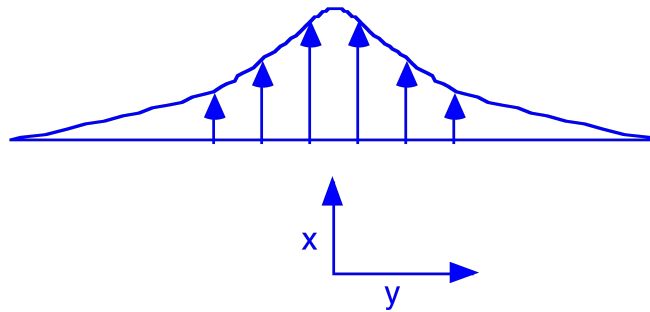
Leal, L. G., "Laminar Flow and Convective Processes," Butterworth-Heinemann, Boston, 1992.

Gebhart, B., *et al.*, "Buoyancy-induced flow and transport," Hemisphere Publ., Washington, 1988.

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Flow due to a Heated Wire: The Theoretical Flow Field

In the vicinity of a line source of energy (in this case the source is a wire carrying electrical current) the temperature is elevated. If the fluid surrounding the source expands upon heating (as is the case for most fluids) the fluid near the wire will have a density less than that of the fluid far from the wire and hence will rise, driving a natural convection circulation pattern. The velocity field produced is approximately as depicted below:



Note that we take the x direction to be *vertical* and the y direction to be horizontal (the z -direction is along the wire and does not figure into a 2-D problem). This is because the flow field may be described as a *boundary layer* (much the same as flow past a flat plate studied in fluids last year), and x is usually taken to be the distance along the boundary. While there isn't a boundary here, there *is* a separation of length scales: the length scale in the horizontal (y) direction is much shorter than the length scale in the vertical (x) direction (as we shall demonstrate), which is really what is required for a boundary layer type flow.

This *Thermal Plume* boundary layer problem is described in a number of transport texts. One reference is L. G. Leal, *Laminar Flow and Convective Transport Processes*, 1992, p. 691. This is the text currently used in the spring semester graduate fluid mechanics course. A complete review of the theory and experiment associated with this problem is provided by Gebhart, B., *et al.*, *Buoyancy-Induced Flow and Transport*, 1988. Both texts have been placed on reserve in the Engineering library.

To derive the equations governing the flow field we need to make a number of assumptions or approximations, which ultimately limit the validity of our solution. We begin with the Boussinesq approximation: we assume that the density fluctuations resulting from the temperature variations are sufficiently small that the only place where they matter is in the buoyancy term of the equations of motion. This is usually a very good approximation for natural convection flows. We also take all other material properties (viscosity, thermal diffusivity, etc.) to be constant (not as good an approximation!). We assume the flow to be strictly 2-D with no variation in the z -direction. This would be correct for steady-state flows with an infinitely long wire: obviously we'll get into trouble if we are too far above a wire of finite length, such that it starts to look like a point source! Finally, we shall ignore all effects of the boundaries - both the side walls (they will induce a finite amplitude recirculation flow) and the top and bottom. Again, this won't be too bad provided that the flow profile is very narrow, and if we are far away from the boundaries!

We will use conservation of mass, momentum, and energy to describe the flow field mathematically. Thus:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = - \frac{\partial P}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \rho g$$

$$\rho \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = - \frac{\partial P}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$

where α is the coefficient of thermal diffusivity and μ is the viscosity (assumed constant). The density profile is given by:

$$\rho = \rho_0 \left(1 - \beta (T - T_0) \right)$$

where β is the coefficient of thermal expansion ($[\Delta V/V]/\Delta T$) and ρ_0 and T_0 are the density and temperature far from the source, respectively.

Far from the wire the temperature and density are constants, and the velocity in the x-direction due to the wire is zero. In this region the pressure distribution is simply that due to hydrostatic pressure variation $p_\infty = p_0 - \rho_0 g x$. If we subtract this off (e.g., let $p = P + p_\infty$), then the momentum equations become:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = - \frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \beta g (T - T_0)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = - \frac{1}{\rho} \frac{\partial P}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)$$

where ν is the kinematic viscosity μ/ρ , and we have divided through by the density.

We also have the boundary conditions:

$$\begin{aligned} u \Big|_{y \rightarrow \pm \infty} &\rightarrow 0 \\ P \Big|_{y \rightarrow \pm \infty} &\rightarrow 0 \\ T \Big|_{y \rightarrow \pm \infty} &\rightarrow T_0 \end{aligned}$$

$$\int_{-\infty}^{+\infty} \left[\rho \hat{C}_p u (T - T_0) - k \frac{\partial T}{\partial x} \right] dy = Q/L$$

where Q/L is the energy dissipated per unit length of the wire. This last condition is a statement of conservation of energy. The energy produced by the electrical dissipation has to go somewhere: at steady state the total integrated energy flux (both convection and conduction) through any plane of constant x must equal the energy released by the wire.

To proceed further we need to render the equations dimensionless. Unlike forced convection problems such as flow past a flat plate, we don't have a reference velocity, temperature, or even a length scale imposed by the boundary conditions. In this case what we shall do is take these quantities to be unknown constants, plug them into the equations, and then choose them so that all the important terms are scaled to be $O(1)$. Thus we take:

$$x^* = \frac{x}{L_x} ; y^* = \frac{y}{L_y} ; u^* = \frac{u}{U_c} ; v^* = \frac{v}{V_c} ; P^* = \frac{P}{P_c} ; T^* = \frac{T - T_0}{T_c}$$

We begin our scaling with the continuity equation. Substituting in and dividing through to render the equation dimensionless yields:

$$\frac{\partial u^*}{\partial x^*} + \left[\frac{V_c L_x}{U_c L_y} \right] \frac{\partial v^*}{\partial y^*} = 0$$

It is *always* the case that, when scaling two dimensional flow problems, both terms of the continuity equation must be of the same magnitude. Thus we choose the quantity in brackets to be unity, and we get:

$$V_c = U_c \left[\frac{L_y}{L_x} \right]$$

e.g., the characteristic velocity in the y -direction is related to that in the x -direction by the ratio of the two length scales. Since we are looking for a boundary-layer type solution, we expect this ratio to be much less than unity.

We now turn to the x -momentum equation. If we substitute back in for V_c , we obtain:

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = - \left[\frac{P_c}{\rho U_c^2} \right] \frac{\partial P^*}{\partial x^*} + \left[\frac{v L_x}{U_c L_y} \right] \left(\frac{\partial^2 u^*}{\partial y^{*2}} + \left[\frac{L_y^2}{L_x^2} \right] \frac{\partial^2 u^*}{\partial x^{*2}} \right) + \left[\frac{\beta g T_c L_x}{U_c^2} \right] T^*$$

The physical mechanisms which must be preserved are the buoyancy source term (that's what drives the entire flow), diffusion of momentum in the y -direction, and the momentum convection term. We will demonstrate in a bit that the pressure gradient term is negligible for natural convection flows. Thus we shall take:

$$\left[\frac{\beta g T_c L_x}{U_c^2} \right] = 1 \quad \text{and} \quad \left[\frac{\nu L_x}{U_c L_y^2} \right] = 1$$

which yields two relations between U_c , T_c , L_x , and L_y . The physical interpretation of the boundary layer length scale (the second relation) is very simple. The time necessary for fluid to be convected in the x direction a distance L_x with velocity U_c is just $t \sim L_x/U_c$. During this time momentum diffuses in the y -direction a distance:

$$L_y = (\nu t)^{1/2} = \left(\frac{L_x \nu}{U_c} \right)^{1/2}$$

which is the appropriate length scale in the y -direction, at least for the momentum boundary layer. Note that diffusion in the x -direction is of order $[L_y^2/L_x^2]$ relative to the other terms in the equation. Terms of this order can be neglected in the boundary layer limit, e.g., when $[L_y^2/L_x^2] \ll 1$.

Next up in our scaling is the y -momentum equation, which we shall use to obtain the correct scaling for pressure variations due to flow. Again substituting in and using the scalings for L_y and V_c derived above we obtain after some manipulation:

$$u^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} = - \left[\frac{P_c L_x}{\mu U_c} \right] \frac{\partial P^*}{\partial y^*} + \frac{\partial^2 v^*}{\partial y^{*2}} + \left[\frac{L_y^2}{L_x^2} \right] \frac{\partial^2 v^*}{\partial x^{*2}}$$

From this equation, we find the appropriate scaling for the pressure variation in the boundary layer is $P_c = \mu U_c / L_x$. We can use this in the x -momentum equation together with the definition of L_y to demonstrate that the pressure gradient term is negligible, e.g.,

$$\left[\frac{P_c}{\rho U_c^2} \right] = \left[\frac{P_c}{\rho U_c^2} \right] \left[\frac{\mu U_c}{P_c L_x} \right] = \left[\frac{L_y^2}{L_x^2} \right] \left[\frac{\nu L_x}{U_c L_y^2} \right] = \left[\frac{L_y^2}{L_x^2} \right] \ll 1$$

Now we look at the energy equation, where we have made the V_c substitution and divided through:

$$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \left[\frac{\alpha L_x}{U_c L_y} \right] \left(\frac{\partial^2 T^*}{\partial y^{*2}} + \left[\frac{L_y^2}{L_x^2} \right] \frac{\partial^2 T^*}{\partial x^{*2}} \right)$$

We have already scaled L_y with U_c and L_x in the momentum equation, however. The scaled energy equation thus becomes:

$$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{\text{Pr}} \left(\frac{\partial^2 T^*}{\partial y^{*2}} + \left[\frac{L_y^2}{L_x^2} \right] \frac{\partial^2 T^*}{\partial x^{*2}} \right)$$

where diffusion in the x-direction is again of order $[L_y^2/L_x^2]$ relative to the other terms in the equation and thus is neglected. It is interesting to note that the Prandtl number $\text{Pr} = \nu/\alpha$ divides the diffusive term in the energy equation. We can't get rid of this ratio by scaling, since it is a material property of the fluid. It always appears in either the momentum or energy equations depending on whether we balance the convective or diffusion terms in the momentum or energy equations. For problems where $\text{Pr} \gg 1$, such as is the case in this experiment, the thermal boundary layer will be much thinner than the momentum boundary layer. In our scaling the momentum boundary layer (e.g., the velocity profile we are trying to measure!) will be $O(1)$ in thickness, while the thermal boundary layer will be $O(1/\text{Pr}^{1/2})$ in thickness. You can actually see the thermal boundary layer as a very thin shimmering sheet due to refractive index variations if you look closely (laser off, please!) at the profile, or the blurring of the particles very close to the wire due to refraction. Note that you can't just throw out the diffusive term for large Pr , as you would not then be able to satisfy the boundary conditions - instead it just leads to a thin thermal boundary layer within the thicker momentum boundary layer! Some references actually rescale y^* in the energy equation to account for this difference in widths, however we shall not do so here.

The final bit of scaling comes from the integral boundary condition. Again inserting the above scalings and dividing through yields:

$$\int_{-\infty}^{+\infty} \left(u^* T^* - \frac{1}{\text{Pr}} \left[\frac{L_y^2}{L_x^2} \right] \frac{\partial T^*}{\partial x^*} \right) dy^* = \left[\frac{Q/L}{\rho \hat{C}_p U_c T_c L_y} \right]$$

Again, we take the quantity in brackets (on the right hand side) to be unity, and ignore the x-diffusion term which is of order $[L_y^2/L_x^2]$. Putting all this together we get three equations for U_c , T_c , and L_y in terms of L_x :

$$\left[\frac{\beta g T_c L_x}{U_c^2} \right] = 1 \quad ; \quad \left[\frac{\nu L_x}{U_c L_y} \right] = 1 \quad ; \quad \left[\frac{Q/L}{\rho \hat{C}_p U_c T_c L_y} \right] = 1$$

After some manipulation, these can be solved to yield:

$$L_y = \left[\frac{\nu^3 \rho \hat{C}_p}{(Q/L) \beta g} L_x^2 \right]^{1/5} \quad ; \quad U_c = \left[\left(\frac{(Q/L) \beta g}{\rho \hat{C}_p} \right)^2 \frac{L_x}{\nu} \right]^{1/5} \quad ; \quad T_c = \left[\frac{(Q/L)^4}{\nu^2 \beta g (\rho \hat{C}_p)^4 L_x^3} \right]^{1/5}$$

and the ratio:

$$\left(\frac{L_y}{L_x}\right)^2 = \left[\frac{v^3 \rho \hat{C}_p}{(Q/L) \beta g} L_x^{-3} \right]^{2/5}$$

It is interesting to note that if the scaling of the equations is done correctly, all the dimensionless -variables- (e.g., the *'ed quantities) should be of O(1). In fact, when the equations are solved numerically, and L_x is chosen to be the distance above the wire, the dimensionless vertical velocity at the centerline of the flow has a value of about 0.9 for $Pr \gg 1$, comfortably O(1). The width of the profile should also be of O(1), with the numerical solution yielding a value of around 1.2 for where the velocity falls to 50% of its maximum. This O(1) property of scaling means that you can make a pretty good guess of the magnitude of the velocity and profile width without ever actually solving the equations! It also suggests that if the dimensionless values you get by solving the equations -aren't- of order unity, then you should probably go back and re-scale things!

While the characteristic scaling values derived above look complex, they arose in a natural way from the balancing of physical mechanisms in the equations of motion. The dimensionless equations, neglecting all terms which are of $O([L_y^2/L_x^2])$, become:

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0$$

$$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{Pr} \frac{\partial^2 T^*}{\partial y^{*2}}$$

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{\partial^2 u^*}{\partial y^{*2}} + T^*$$

$$\int_{-\infty}^{+\infty} u^* T^* dy^* = 1$$

In the above derivation we have made a number of approximations to simplify the equations. In no particular order, these are:

- 1) The Boussinesq approximation, $\beta T_c \ll 1$; $\frac{1}{\mu} \frac{\partial \mu}{\partial T} T_c < 1$ (no viscosity variation)
- 2) Two-Dimensional flow, $[L_y^2/L_z^2] \ll 1$ (L_z is the length of the wire)
- 3) Steady-state flow
- 4) The boundary layer approximation, $[L_y^2/L_x^2] \ll 1$.
- 5) An infinite medium (no wall effects)

It is interesting to note that all parameters except the Prandtl number have been scaled out of these equations, and yet we have not had to specify L_x , the characteristic length scale in the x-direction. This occurs often in boundary-layer problems (problems where there is a large

separation in the length scales in flow and cross-streamline directions), and results in a similarity solution.

A key result of the above scaling analysis is the dependence of the characteristic length scale in the y-direction and the characteristic velocity on L_x . We find that U_c varies as $L_x^{1/5}$, and L_y varies as $L_x^{2/5}$. This suggests that the velocity will increase as $x^{1/5}$ power as we move up above the wire, and that the width of the velocity profile will increase as $x^{2/5}$. We will make use of these scalings in developing our similarity solution to the transport equations.

The flow pattern produced by a line source is two dimensional, thus we may define a stream function ψ :

$$u^* = \frac{\partial \psi^*}{\partial y^*} ; v^* = -\frac{\partial \psi^*}{\partial x^*}$$

In terms of the streamfunction the equations become:

$$\begin{aligned} \psi_y^* \psi_{xy}^* - \psi_x^* \psi_{yy}^* &= \psi_{yyy}^* + T^* \\ \psi_y^* T_x^* - \psi_x^* T_y^* &= \frac{1}{Pr} T_{yy}^* ; \int_{-\infty}^{+\infty} \psi_y^* T^* dy^* = 1 \end{aligned}$$

where subscripts denote partial derivatives with respect to the argument. The similarity transform is given by:

$$\begin{aligned} \psi^* &= x^{*3/5} f(\eta) ; T^* = x^{*-3/5} g(\eta) ; \eta = \frac{y^*}{x^{*2/5}} \\ \text{with } u^* &= \psi_y^* = x^{*1/5} f'(\eta) \\ \text{and } v^* &= -\psi_x^* = -x^{*-2/5} \left(\frac{3}{5} f(\eta) - \frac{2}{5} \eta f'(\eta) \right) \end{aligned}$$

We may also find the velocity derivative of interest:

$$\frac{\partial v^*}{\partial y^*} = -\psi_{xy}^* = -x^{*-4/5} \left(\frac{1}{5} f'(\eta) - \frac{2}{5} \eta f''(\eta) \right)$$

and the resulting ODE's:

$$f''' = -g + \left[\frac{1}{5} (f')^2 - \frac{3}{5} f f'' \right] ; g' = -Pr \frac{3}{5} f g$$

where primes denote derivatives with respect to the similarity variable η . We also have the

transformed boundary conditions:

$$f(0) = f''(0) = 0$$

$$f' \rightarrow 0 \text{ as } \eta \rightarrow \infty$$

$$\int_{-\infty}^{+\infty} f' g d\eta = 1$$

The boundary conditions at $\eta = 0$ are symmetry conditions: both v^* and $\frac{\partial u^*}{\partial y^*}$ must vanish at the centerline. These equations may be solved numerically as a function of the Prandtl number. The procedure for solving the equations numerically is to set them up as a system of four first order differential equations for f, f', f'' , and g . We have initial conditions for f and f'' , but we must determine initial conditions for f' (the dimensionless velocity) and g (the dimensionless temperature) at the centerline. We do this by using the shooting method, guessing values of $f'(0)$ and $g(0)$ and iterating until both the boundary condition $f'(\infty) = 0$ and the integral boundary condition are satisfied. The integral boundary condition may be tracked by simply adding its integral as a fifth first order differential equation in the integration process.

Plots of the solution for $Pr = 68$ are given in the following diagram. Note that the dimensionless velocities have been scaled by the dependence on x^* predicted by the similarity solution. The program which generated this diagram is available at the URL:

<http://www.nd.edu/~dtl/cheg459/pivexperiment>

You should be prepared to discuss these theoretical results and, in particular, be able to estimate where the approximations which led to the solution will break down.

Stratification Issues

One assumption which is often violated in experiments is that the fluid is uniform: that there is no vertical density stratification. There are many possible causes of stratification. For example, the wire itself locally heats the fluid, and the warm fluid flowing upwards eventually leads to a vertical stratification in temperature. The mixing of water and glycerin is exothermic, causing the temperature of the mixed solution to be a few degrees higher than the ambient temperature. While (if well mixed!) this does not directly produce stratification, the now colder walls exposed to the lab air cause a cold draft that pools down on the bottom of the container. Incomplete mixing causes density stratification due to composition (a slightly higher glycerin concentration in the fluid near the bottom, and the reason why you need to mix thoroughly!). The effect of all these sources of stratification is fairly straightforward: since the flow due to the heated wire draws fluid upwards, the flow is resisted by any vertical density stratification. This is evident in a couple of features of the profile: First, the profile is slightly "pinched in" in the y -direction, with negative velocities in the vicinity of the primary vertical upward flow. This is caused by the denser fluid flowing back down (sort of the return flow of a fountain). Second, it massively suppresses the flow at very low power levels. For really low wattages the temperature

differential produced by the wire is unable to overcome the vertical density stratification at all, and the flow basically disappears. Finally, the stratification is most easily seen if you simply turn off the power and continue to watch the flow. If there is no stratification, the flow will simply slow down and stop (after a short interval). If there is vertical stratification, however, the denser fluid which was drawn upwards by the flow will "rebound" through the center, producing an inverse flow of lower magnitude.

There are several ways we can deal with stratification. First, you should try to avoid it in the first place. Using the information from the Dow website, you can determine the exothermic gain in temperature upon mixing. By pre-cooling the water to an appropriate degree, you can prepare a solution which, when mixed, matches the laboratory temperature. Second, you will need to mix your solution thoroughly, and if stratification occurs simply pour it back into your container and shake it up again. You can also estimate the temperature gradient produced by your heating of the fluid over time from an energy balance. Finally, you can try to measure the temperature profile in the vertical direction using the thermocouple (this is a bit tricky, as the gradient is really small!), and experimentally determine the vertical temperature gradient. This can be combined with the theory to determine a figure of merit to see where stratification will be an issue.

To see how to do this theoretically, consider what happens in the absence of stratification (and for which the self-similar solution is valid). We are interested in the fluid density profile at a given vertical position above the wire. The density profile will be the result of two effects: the convection of the heated fluid upward by the flow (essentially that provided by the self-similar solution to the temperature distribution), and the density profile due to the stratified fluid being drawn up to higher vertical positions (x). The stratification will be a problem when the integrated buoyancy due to these two effects becomes comparable. The first is just the integral of the temperature distribution over y :

$$\text{Plume Buoyancy} = \int_{-\infty}^{+\infty} \rho \beta g (T - T_0) dy = \rho \beta g T_c L_y x^{-1/5} \int_{-\infty}^{+\infty} g d\eta$$

Note that this decreases as we move up above the wire. This is because, even though the thermal profile (much narrower than the velocity profile at large Pr) is increasing with x , the temperature is decreasing. For large Pr the thermal plume is much narrower than the velocity profile. This means that over the entire thermal plume, the velocity is approximately that of the centerline. Thus the integral boundary condition (energy balance) used in self-similar solution can be used to estimate the integral of g alone:

$$1 = \int_{-\infty}^{+\infty} f' g d\eta \approx \int_{-\infty}^{+\infty} f'(0) g d\eta \quad \therefore \int_{-\infty}^{+\infty} g d\eta \approx \frac{1}{f'(0)}$$

The buoyancy due to density stratification depends on both the density gradient and the distance we are displacing the fluid. If the former is due to a vertical gradient in bath temperature $\partial T_0/\partial x$, we have the buoyancy integral:

$$\text{Stratification Buoyancy} = \int_{-\infty}^{+\infty} -\rho \beta g \frac{\partial T_0}{\partial x} (x - x_0) dy$$

where x_0 is the vertical position that a fluid particle at (x,y) originated from (x_0, ∞) due to the flow field. We may determine this by using the streamfunction. Remember that the streamfunction is constant along a streamline. Thus, we have that:

$$\psi|_{x,y} = \psi|_{x_0, \infty}$$

Using the self-similar solution for the streamfunction we have the simple expression for x_0 :

$$x_0^{*3/5} f(\infty) = x^{*3/5} f\left(\frac{y^*}{x^{*2/5}}\right)$$

or the equivalent displacement in the x-direction:

$$x^* - x_0^* = x^* \left(1 - \left(\frac{f(\eta)}{f(\infty)}\right)^{5/3}\right)$$

The ratio of this stratification induced buoyancy to that produced by the heated wire is thus simply:

$$\text{Ratio} = \frac{\frac{\partial T_0}{\partial x}}{\frac{T_c}{L_x}} x^{*8/5} \int_{-\infty}^{+\infty} f'(0) \left(1 - \left(\frac{f(\eta)}{f(\infty)}\right)^{5/3}\right) d\eta$$

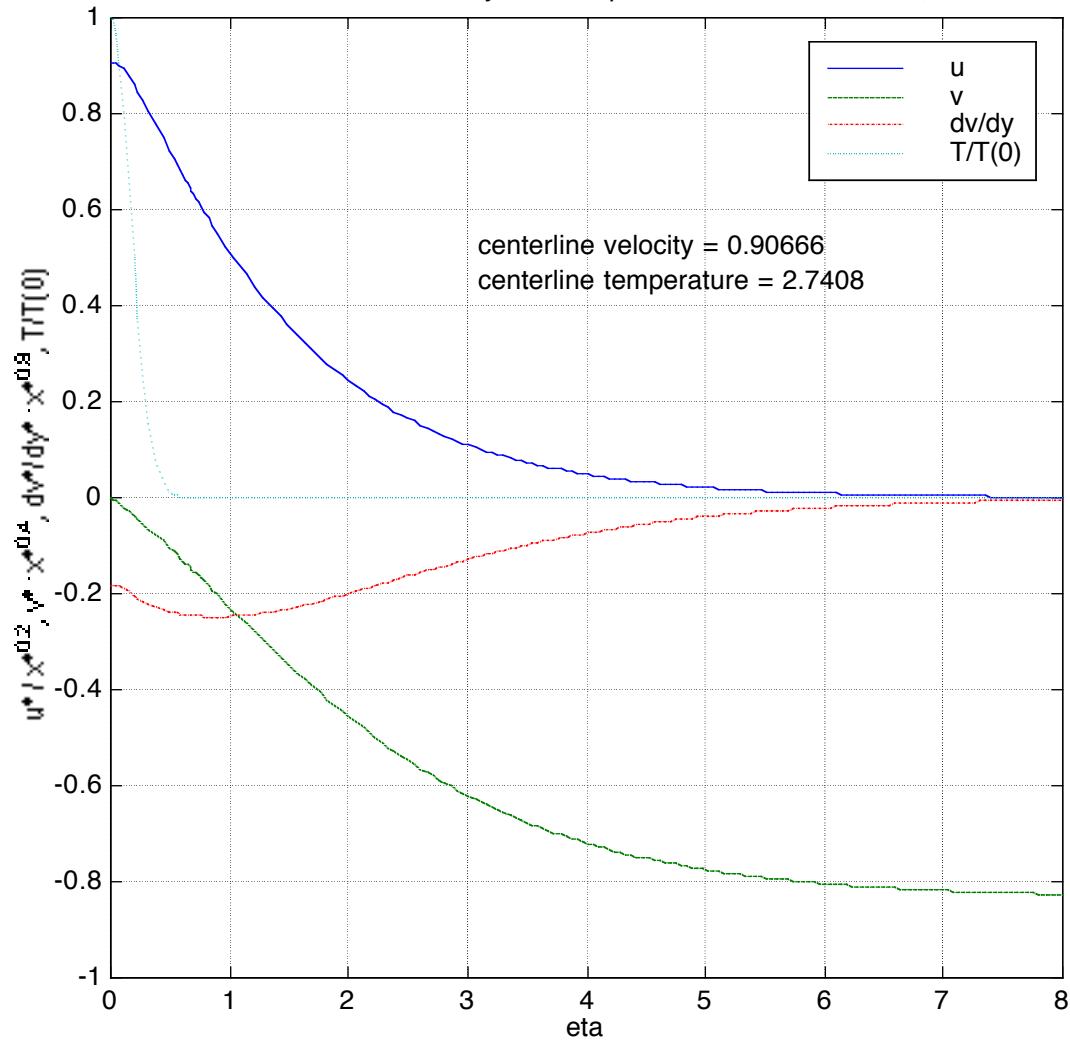
Dimensionally you can see that the ratio increases with height above the wire (to the 1.6 power in x), and is proportional to the ratio of the thermal stratification gradient and T_c/L_x . The integral is an $O(1)$ function of Pr . For large Pr (even for pure water, $Pr \sim 7$) it has a value of about 3.

This suggests conditions under which stratification is likely to be significant and observable: low power, far above the wire, and large vertical temperature gradients. A vertical gradient which approaches

$$\frac{\partial T_0}{\partial x} \sim \frac{1}{3} \frac{T_c}{L_x} x^{*-8/5}$$

will likely yield stratification effects. You should compare your experimental results to this criteria and see if it can be used to describe some of your deviations from theoretical predictions.

Plot of Dimensionless Velocity and Temperature for a Heated Wire, Pr = 68



Detailed Experimental Instructions

1. **Precalculation of Mixture Composition**

In order to avoid stratification, it is necessary to get the mixture at the same temperature as the lab. Using the provided material property programs, calculate the required temperature of the water so that the heat of mixing is “cancelled out”. This temperature can be most easily achieved with a mixture of tap water and ice water – you just have to calculate how much to use! Measure the room, glycerin, and tap water temperatures and adapt the material property programs provided on the website to determine desired compositions.

2. **Preparing the Mixture**

Measure out a 3L solution of water and glycerin at the desired composition and add approximately 0.04g of the chalk particles. Mix the solution well by shaking it for about 5 minutes in a Nalgene container (water and glycerin do not mix easily, so a lot of mixing is required). Next, transfer the solution to the glass tank. It will take a few minutes for the bubbles to clear. Be sure to scrape off any bubbles that are attached to the front or side of the tank! Be sure to clean off the surfaces of the tank to get good images!

3. **Placing the Wire**

The cradle holding the heated wire locks into the two pins at the bottom of the tank (a bit tricky sometimes). After it is locked in, set the tank on the stage and connect the wires to the + and – terminals of the power supply. The pdf manual for the power supply is on the computer desktop. Don't forget to measure the length of the wire before inserting it!

4. **Adjusting Camcorder Position**

Place the camcorder on the tripod (if it's not there already) and adjust the height to match the level of the laser if necessary. Only the box holding the solution should be moved after this point. Rather than using the micrometer on the translation stage to crank it up and down, it is much easier to insert or remove the ½” (1.26cm) thick plastic plates. This is about the right distance to get slightly overlapping laser sheet imaging regions.

5. **Turn the Camcorder On**

To turn the camcorder on, switch the dial with the green button in the back to “camera”. Do *not* put it into “memory” mode (e.g., pushing the dial a bit too far), as that changes the zoom and makes the images darker. An exceedingly annoying feature of the camera is that it puts itself into “demo mode” after about 6 minutes. Thus, before you are “ready to rock”, turn it off and back on again to reset its counter.

6. **Focus and Zoom**

Autofocus works fine for this experiment (if you aren't zoomed in too far). Using the remote control (it is in the top drawer of the table), adjust the zoom to the desired amount. In general if you are close to the wire you want to be zoomed in, but far above the wire you want to be zoomed out. The camera has manual focus too, but it is a bit tricky to use. The pdf of the camera manual is located on the computer desktop.

7. Setting up the Computer

At the beginning of the period you should set up a folder on the computer that will contain your images and your programs. After creating such a folder, download all of the programs from the experiment website to it. Most of these programs will be used without modification, but some you will want to adapt for your purposes. You can do both at once by double clicking on the “programs.zip” file on the desktop and renaming the folder it creates.

8. Capturing Film Clips

Because there is a direct link to the computer (the camera should be plugged into the firewire port of the computer), no tape is needed. Remember to label every experiment immediately after recording and record the name in your lab notebook along with experimental conditions! With the camera is connected to the computer, open iMovie from the menu bar on the computer monitor. Below the iMovie viewing screen is a camcorder icon and a magnifying glass icon. Switch the selector to the camcorder. When the camera is ready (and you want to start capturing), click import to start streaming the video to the computer. Click “Done” when you are finished.

9. Calibration

With the flow system exposed to room light, insert a ruler into the tank in the sheet of laser light and capture a brief clip for calibration purposes. You should also measure the height of the wire above the bottom of the tank with the same ruler, as analysis requires determining the location of the image centers relative to the wire, not the tank itself. It is useful to insert a sheet of white paper behind the tank to make it easier to see the markings on the ruler (but take it out after you are done!). You will need to use a second ruler to measure the width of the one inserted (needed for horizontal calibration). Calibration must be done any time you have adjusted the zoom.

10. Adjust Lighting

Hang the black curtain over the camcorder and cover the front of the experiment area. You should now be ready to measure the velocity profile. Make sure the flow field has settled down after inserting the ruler before your measurements!

11. Preparing Movies for Analysis

Once the images have been collected, up to 99 frames need to be saved for analysis with Matlab. Find the “event” corresponding to your recording and click on it. Using the cursor, select about 3 seconds of the image which contains particles traveling through the region of interest (remember, you are measuring particle velocities, so if you don’t see a particle somewhere, you won’t be able to measure the fluid velocity there either!). Grab these selected frames and drag them down to the bottom bar (having deleted whatever was there already).

12. Converting Movies to Image Sequences

Once the desired frames are the only thing in the editing area, go to the “Share” menu and choose “export using Quicktime”. At the top of the window that appears click on the Quicktime icon. In the “compress movie for:” dialog box choose “expert settings” and then

click share. Choose a name for the frames. This will be the base name of individual images, and must be different for every clip you wish to analyze. You also need to choose a directory. The export settings should be correct, but make sure that in the “Export” dialog box it reads “movie to image sequence” and that under “options” the file setting is “jpeg.” Then click save. Save these images to the directory you have previously created, and which has your Matlab analysis programs in it.

13. Storing and Analyzing Files

Once the files have been saved, analyze each set of images using `pivanal.m` and `pivplot.m`, running the calibration program `calibration.m` as necessary. Note that you need to change the pixel calibration settings every time you move or adjust the camera. Vertical motion of the tank shouldn't change the calibration, and you can use the micrometer on the vertical adjustment slide for the tank to determine the change in vertical position of the wire between successive sets of images. Thus, you can avoid reinsertion of the ruler for calibration between each set - and the resulting disturbance of the flow field - allowing faster data acquisition. Be sure to write down the position, however!

14. Analyzing Multiple Sets of Images

You will need to analyze multiple sets of images to build up a complete picture of the flow field. This can be easily done after all of them have been analyzed individually, as the program `pivplot.m` saves the position and velocity data under a filename corresponding to the names you chose for each image sequence. You will need to write an analysis program which loads each of these data files in sequence and appends the data to a master list of positions and velocities. At this point it is simple to produce a quiver plot of the velocities over the entire tank, or to select data points within restricted ranges in the vertical direction to evaluate profile evolution. Note that you could collapse all the data onto a single curve (that is, if the theoretical model is perfect!) by rendering velocities and corresponding horizontal positions dimensionless, scaling them appropriately with x , and then plotting the scaled velocities vs. the similarity variable. Looking through the program `pivplot.m` should give you some ideas what sorts of plots would be useful and interesting in comparing theory to experiment. Be creative!

15. Save Your Work

Don't count on any of the information you leave on the computer to remain there from week to week. Thus you should save any of the data files you want to keep for preparing your report on your own computer. In particular, you may wish to save the particle link files and particle velocity files. The program `pivplot.m` does not require any of the image files to run, just the particle link data generated by `pivanal.m` and the calibration information. You thus do not need to save the image files (although you may wish to do so).

Image Trouble Shooting: Pivanal.m Parameter Descriptions

npics

This is the number of frames you want to analyze. You should determine the number of frames you have captured and set this variable to that value. Note that you have to have less than 99

frames transferred (naming conventions change if you have more than 99), and can't analyze more frames than you have.

threshold

The threshold sets a minimum brightness level to determine where the particles are. A threshold of 50 is the default value. For most images, this should track enough particles to give a meaningful profile. If the images are somewhat blurry or there are holes in your velocity profile, lower the threshold by increments of 5, until a better collection of particles are found. Lowering the threshold too much may clutter your velocity profiles.

gain

Another way of increasing the number of points is to lower the gain. Lowering the gain will allow weaker lit particles to survive the filtering of the image. The threshold, however, has a greater impact on the image than the gain.

minframes

A particle must be tracked for a minimum number of frames in order to have its velocity calculated. This parameter is defined in pivanal.m as

```
minframes=min(npics/2-.1,7.5)
```

The velocities are sometimes not calculated correctly at higher power where the velocity is higher. In order to fix improper tracking and velocities, change the value in the minframes expression to a new value and run the analysis again. A larger number will require a spot to be in more frames.

maxdist

The maximum distance that a particle can travel, and still be analyzed as the same particle, is defined by the maxdist variable. The purpose of this variable is to ensure that two particles don't get mistaken as the same particle. This is mainly a concern with low velocities. At large velocities, however, the particles start traveling out of this radius due to their speed. Increase the maxdist parameter if this appears to be the problem. If a few velocities appear too high, the program may have mistaken a new particle as an old one and the parameter may need to be decreased.

Corollary: If you would like to investigate the horizontal velocities and believe that the program is not tracking the velocities correctly, change the xfactor value. The xfactor is the ratio of the maximum distance a particle can travel in the vertical direction compared to the horizontal direction. This should not be much of an issue.