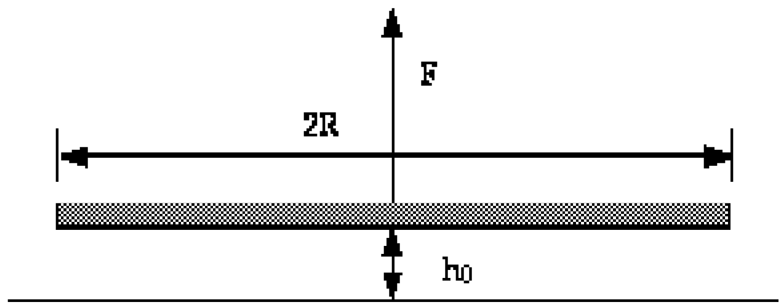
CHEG 544 Transport Phenomena I
Second Hour Exam

Closed Books and Notes

Problem 1). (20 points) Lubrication: In the undergrad fluids course last fall, I demonstrated how the initial separation distance controls the detachment of a disk of radius R from a plane, as depicted below.



If the upward force on the disk is given by F (ignore all buoyancy and gravitational effects!), use lubrication theory to determine how long a disk with initial separation $h_0 << R$ takes to detatch from the plane. Detatchment occurs when the separation distance becomes much greater than the initial separation, e.g., $R >> h(t) >> h_0$. You may find the r momentum equation useful to get the pressure gradient:

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r}$$
$$+ \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\, v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right] + \rho\, g_r$$

Remember that in lubrication flow (at least for this problem) only a couple of terms survive!

Problem 2). (20 points) Creeping Flow/Index Notation: Consider a sphere of radius a undergoing solid body rotation with angular velocity Ω_i in an unbounded fluid at rest. Using index notation, determine the disturbance velocity produced by the sphere.

Problem 3). (20 points) Consider the two-dimensional problem depicted below. Two belts separated by an angle 2α are in motion, such that each has a velocity of +U in the radial direction. We wish to determine the velocity profile using a streamfunction formulation. The velocities are given by:

$$u_\theta = -\frac{\partial\psi}{\partial r} \qquad ; \qquad u_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta}$$

Recall that the general expression for a separable streamfunction in the cylindrical geometry is given by:

$$\psi_\lambda = r^{\lambda} f_\lambda(\theta)$$

where, in general, we have:

$$f_\lambda(\theta) = A_\lambda \sin(\lambda\theta) + B_\lambda \cos(\lambda\theta) + C_\lambda \sin((\lambda\text{-}2)\theta) + D_\lambda \cos((\lambda\text{-}2)\theta)$$

We also have the repeated root special cases:

$$f_0(\theta) = A_0 + B_0\,\theta + C_0 \sin(2\theta) + D_0 \cos(2\theta)$$

$$f_1(\theta) = A_1 \sin(\theta) + B_1 \cos(2\theta) + C_1\,\theta \sin(\theta) + D_1\,\theta \cos(\theta)$$

$$f_2(\theta) = A_2 + B_2\,\theta + C_2 \sin(2\theta) + D_2 \cos(2\theta)$$

Using the above relations, solve for the streamfunction and velocity distribution. Hint: Think about how the radial velocity and streamfunction have to depend on r, and about symmetry relations for u_r and Ψ in θ.