Problem 1). (20 points) Lubrication: In the undergrad fluids course last fall, I demonstrated how the initial separation distance controls the detachment of a disk of radius R from a plane, as depicted below.

If the upward force on the disk is given by F (ignore all buoyancy and gravitational effects!), use lubrication theory to determine how long a disk with initial separation $h_0$ << R takes to detach from the plane. Detachment occurs when the separation distance becomes much greater than the initial separation, e.g., $R >> h(t) >> h_0$. You may find the $r$ momentum equation useful to get the pressure gradient:

$$\rho \left( \frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_r}{r} \frac{\partial v_\theta}{\partial \theta} + v_z \frac{\partial v_r}{\partial z} \right) = - \frac{\partial p}{\partial r}$$

$$+ \mu \left[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (r v_r) \right) + \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2} \right] + \rho g r$$

Remember that in lubrication flow (at least for this problem) only a couple of terms survive!

Problem 2). (20 points) Creeping Flow/Index Notation: Consider a sphere of radius a undergoing solid body rotation with angular velocity $\Omega_i$ in an unbounded fluid at rest. Using index notation, determine the disturbance velocity produced by the sphere.
Problem 3). (20 points) Consider the two-dimensional problem depicted below. Two belts separated by an angle $2\alpha$ are in motion, such that each has a velocity of $+U$ in the radial direction. We wish to determine the velocity profile using a streamfunction formulation. The velocities are given by:

\[
\begin{align*}
qu &= -\frac{\partial \psi}{\partial r} ; & qu &= \frac{1}{r} \frac{\partial \psi}{\partial \theta} 
\end{align*}
\]

Recall that the general expression for a separable streamfunction in the cylindrical geometry is given by:

\[
\psi_{\lambda} = r \lambda \hat{f}_{\lambda}(\theta)
\]

where, in general, we have:

\[
\hat{f}_{\lambda}(\theta) = A_{\lambda} \sin (\lambda \theta) + B_{\lambda} \cos (\lambda \theta) + C_{\lambda} \sin ((\lambda - 2) \theta) + D_{\lambda} \cos ((\lambda - 2) \theta)
\]

We also have the repeated root special cases:

\[
\begin{align*}
f_0(\theta) &= A_0 + B_0 \theta + C_0 \sin(2 \theta) + D_0 \cos(2 \theta) \\
f_1(\theta) &= A_1 \sin(\theta) + B_1 \cos(2 \theta) + C_1 \theta \sin(\theta) + D_1 \theta \cos(\theta) \\
f_2(\theta) &= A_2 + B_2 \theta + C_2 \sin(2 \theta) + D_2 \cos(2 \theta)
\end{align*}
\]

Using the above relations, solve for the streamfunction and velocity distribution. Hint: Think about how the radial velocity and streamfunction have to depend on $r$, and about symmetry relations for $u_r$ and $\psi$ in $\theta$. 

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**Diagram:**

- Two belts separated by an angle $2\alpha$.
- Each belt moving with velocity $+U$ in the radial direction.
- The angle between the belts is $2\alpha$.

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**Diagram Notes:**

- The belts are depicted with arrows indicating their direction of motion.
- The angle between the belts is marked as $2\alpha$.
- The velocity $+U$ is shown in the radial direction for each belt.