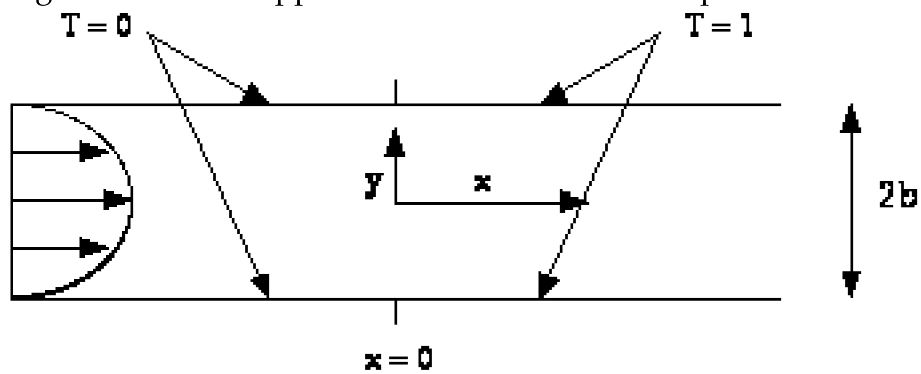
CHEG 544 Transport Phenomena I
Final Exam

Closed Books and Notes

Problem 1). Most of this semester we have examined the asymptotic solution to comple x problems. In this problem you will examine convective heat transfer in pressure driv en flow through a channel. Suppose we have the channel depicted below:



For all $x < 0$ the walls are maintained at a temperature $T = 0$, and for $x > 0$ at a temperat ure $T = 1$. The fluid velocity at the centerline is U, the width of the channel is 2b, and the thermal diffusivity is α. If we assume constant properties everywhere, the problem is governed by the differential equation:

$$u_j \frac{\partial T}{\partial x_j} = \alpha \frac{\partial^2 T}{\partial x_j^2}$$

where the velocity is just unidirectional Poiseuille flow. Close to the entrance to the hea ted section (but not too close!) the temperature profile admits a self-similar boundary la yer solution near the walls. I want you to solve the following problems:

a. Scale the energy equation for this region, and explicitly show over what domain in x a boundary layer solution would be expected to occur (upper and lower limits, please!). Under what condition will there be -no- domain of validity?

b. Determine the similarity rule and variable in canonical form, and obtain the resulting ODE and boundary conditions.

c. Solve the ODE - you may leave the final answer in terms of integrals if you wish.

Problem 2). In a recent homework you showed that boundary layer growth was retarded by an accelerating flow. In this problem we look at the opposite case: a decelerating flow. Consider flow past a flat plate where the x-velocity outside the boundary layer is given by $u_e = \lambda\, x^{-1/2}$.

a. Render the Navier-Stokes equations dimensionless for this problem and determine the conditions under which a boundary layer description is valid.

b. Using simple affine stretching, show that the boundary layer equations admit a self-similar solution, and determine the rate of boundary layer growth with x. Obtain the similarity rule, variable, and resulting ODE with boundary conditions.

Problem 3). Lubrication: For homework you solved for the closing rate of a hinged plate. Here we consider a very similar problem. A plate (ignore gravity this time) has a force F applied at the outside edge, pushing it toward the plane. The pressure distribution resulting from the squeeze flow makes the plate want to separate from the plane at the hinge. Using lubrication theory, show that the force F_h exerted by the hinge on the plate for small separation angles is exactly 3F, e.g., three times the closing force - which is why hinges tend to pop open! Hint: Remember that both torque and force must balance.