CHEG 544 Transport Phenomena I
Second Hour Exam

Closed Books and Notes

Problem 1). (20 points) Boundary layer flow past a flat plate at zero incidence is govern ed by the boundary layer equation:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}$$

with boundary conditions:

$$u\,|_{y \to \infty} = u\,|_{x=0} = U \;\;,\;\; u\,|_{y=0} = v\,|_{y=0} = 0$$

where ν is the kinematic viscosity, and u and v are the velocities along the plate (x direct ion) and normal to the plate (y direction), repectively. In terms of the streamfunction w e have the equivalent expression:

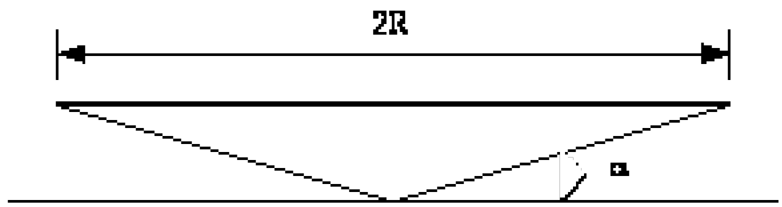
$$\psi_y \psi_{xy} - \psi_x \psi_{yy} = \nu\, \psi_{yyy}$$

and:

$$\psi_y\,|_{y \to \infty} = \psi_y\,|_{x=0} = U \;\;,\;\; \psi_y\,|_{y=0} = \psi_x\,|_{y=0} = 0$$

Render these equations dimensionless and show that they admit a self-similar solution a ccessible through simple affine stretching. Obtain the similarity rule and variable in can onical form, as well as the new ODE and boundary conditions. This will be the Blasius e quation for flow past a flat plate derived in class.

Problem 2). (20 points) Lubrication: A cone of radius R and angle α is initially in contact with a plane as depicted below.



Using lubrication theory, determine the force F necessary to pull the cone off the plane with some velocity V in the limit of small α. You may find the r momentum equation u seful to get the pressure gradient:

$$\rho \left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z} \right) = - \frac{\partial p}{\partial r}$$

$$+ \mu \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (r\, v_r) \right) + \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2} \right] + \rho\, g_r$$

Remember that in lubrication flow (at least for this problem) only a couple of terms sur vive!

Problem 3). (20 points) Consider the two-dimensional problem depicted below. Fluid d rains from a trough with interior angle 2a at a rate Q/W where W is the extension of th e trough in the third dimension. We wish to determine the velocity profile using a strea mfunction formulation. The velocities are given by:

$$u_\theta = -\frac{\partial \psi}{\partial r} \qquad ; \qquad u_r = -\frac{1}{r}\frac{\partial \psi}{\partial \theta}$$

Recall that the general expression for a separable streamfunction in the cylindrical geom etry is given by:

$$\psi_\lambda = r^{\lambda} f_\lambda(\theta)$$

where, in general, we have:

$$f_\lambda(\theta) = A_\lambda \sin(\lambda\theta) + B_\lambda \cos(\lambda\theta) + C_\lambda \sin((\lambda\text{-}2)\theta) + D_\lambda \cos((\lambda\text{-}2)\theta)$$

We also have the repeated root special cases:

$$f_0(\theta) = A_0 + B_0\,\theta + C_0 \sin(2\theta) + D_0 \cos(2\theta)$$
$$f_1(\theta) = A_1 \sin(\theta) + B_1 \cos(2\theta) + C_1\,\theta \sin(\theta) + D_1\,\theta \cos(\theta)$$
$$f_2(\theta) = A_2 + B_2\,\theta + C_2 \sin(2\theta) + D_2 \cos(2\theta)$$

Using the above relations, solve for the streamfunction and velocity distribution. Hint: Think about how the radial velocity and streamfunction have to depend on r, and deter mine the symmetry conditions in θ!