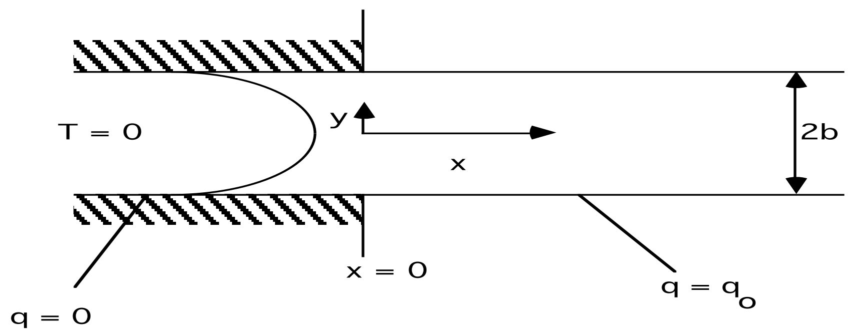
CHEG 544 Transport Phenomena I
Final Exam

Closed Books and Notes

Problem 1). Many analogies exist between energy and momentum transfer. In this pro blem we will look at the temperature distribution acquired by a fluid in a channel as it p asses from an insulated region through a region where there is a constant heat flux into the fluid at the walls. Throughout this problem we will assume that the velocity distrib ution is fully developed (parabolic). The differential equation governing the temperatur e distribution and the boundary conditions are given below.



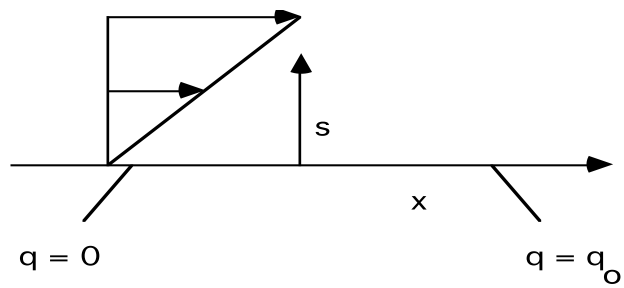
$$u_i \frac{\partial T}{\partial x_i} = \alpha \frac{\partial^2 T}{\partial x_i^2}\ , \quad -k\ n_i \frac{\partial T}{\partial x_i}\bigg|_{\substack{x_2 = \pm b \\ x_1 < 0}} = 0\ , \quad -k\ n_i \frac{\partial T}{\partial x_i}\bigg|_{\substack{x_2 = \pm b \\ x_1 \geq 0}} = q_0 \qquad \begin{array}{l} x_1 \equiv x \\ x_2 \equiv y \end{array}$$

a. (5 points) Render the problem and boundary conditions dimensionless. Show, by re ndering x dimensionless such that x-convection balances y conduction, that for sufficient ly high velocities conduction in the x-direction is negligible.

b. (10 points) Neglecting conduction in the x-direction solve for the asymptotic tempera ture distribution far down the channel.

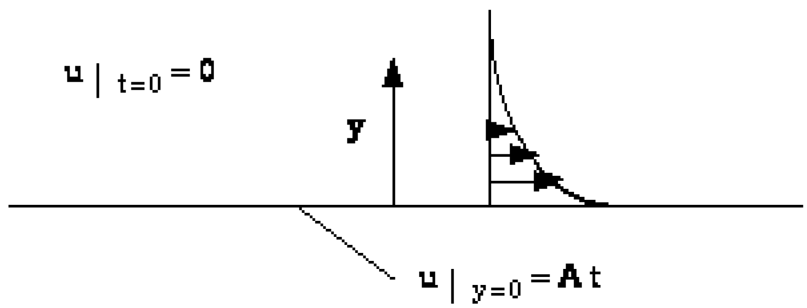
c. (10 points) Obtain the solution for finite distances down the channel via separation of variables. Obtain an explicit equation for the coefficients of this series solution, but do n ot evaluate the resulting integral.

d. (10 points) For small values of x (and values of y close to the wall) it is simpler to solv e for the temperature distribution via a self-similar solution to the boundary layer equat ion. To solve for the temperature profile in this region, let us consider the flow at the u pper wall and define a coordinate s=b-y. Show that if we neglect conduction in the x dir ection (as before) and approximate the velocity profile by the linear shear flow $u=\gamma\, s$ (its limiting form as $s \rightarrow 0$) then the problem will admit a similarity solution. Obtain the si milarity rule, similarity variable, and transformed ODE with corresponding boundary c onditions (in canonical form please) but do not solve the resulting problem. How does the temperature at the wall and the thickness of the thermal boundary layer vary with x?



e. (5 points) Explicitly determine the domain of validity of the solutions to parts b-d. U nder what conditions does the domain of validity of the solution in part d entirely vanis h?

Problem 2). (20 points) An infinite plate (think unidirectional flow here) is accelerated fr om rest with a velocity given by U = A t. Determine the shear stress at the plate to with in some unknown multiplicative order one constant.

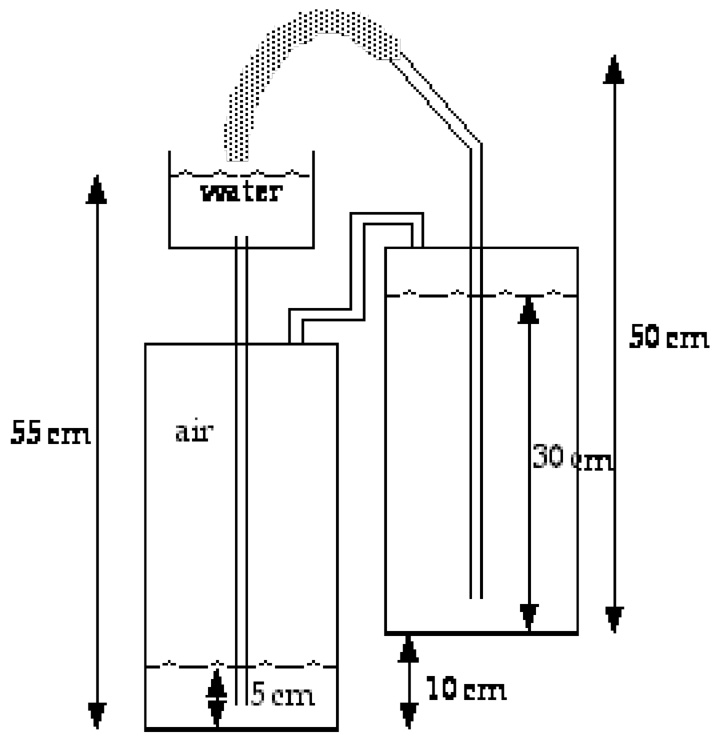


Problem 3). (20 points) A couple of weeks ago Keith Moffatt gave a fascinating lecture on magneto-electro-hydrodynamic migration of particles. He showed that if particles were sufficiently small that inertial forces were negligible, then the migration velocity of non-conducting particles was *Bilinear* in the current density **J** (a physical vector) and the magnetic field **B** (a pseudo vector). Using this observation, prove that the migration ve locity of a non-conducting sphere is always orthogonal to both the magnetic field and t he current, and that the rotational velocity of such a sphere is identically zero.

Problem 4). (20 points) Hero's Fountain. In CHEG 355 we demonstrated Hero's Fount ain, attributed to Hero of Alexandria a couple of millenia ago. In this problem we analy ze its performance.

a). Neglecting all frictional losses, what is the exit velocity and flow rate of the fountain? All pipes are 1cm ID smooth tubes.

b). Modify your answer by accounting for the head losses in the pipes and fittings. Cor relations for friction factors in pipes and fittings are given below. You may take the tot al length of pipe to be 100 cm.



$$h_L = \frac{\langle u \rangle^2}{2g} \sum K + 4 \; f_f \frac{L}{D} \frac{\langle u \rangle^2}{2g}$$

$$f_f = \frac{16}{Re} \; ; \; \text{Re} < 2100 \qquad f_f \approx \frac{0.0791}{Re^{1/4}} \; ; \; 3000 < \text{Re} < 10^5$$

$$\frac{1}{\sqrt{f_f}} = 4.0 \, log_{10} \left(Re \quad f_f \right) - 0.40 \; ; \; \text{Re} > 3000$$

Fitting	K value
sudden contraction	0.45
sudden expansion	1.0
45° elbow	0.35