CHEG 544 Transport Phenomena I
First Hour Exam

Closed Books and Notes

1). (10 points) In class we discussed the drift of a stresslet (a point singularity in low Reynolds number flows) due to the presence of a plane. Stresslets describe the far-field ($r/a >> 1$) disturbance velocity produced by any force-free and torque-free particles in a shear flow, examples being neutrally buoyant drops, rigid particles, and cells. A stresslet is specified by the second order physical tensor S_{ij} which is symmetric ($S_{ij} = S_{ji}$) and traceless ($S_{ij}\delta_{ij} = 0$). The drift velocity u_i of the stresslet (e.g., the particle) is proportional to S_{ij} for creeping flows, and will be a function of the unit normal n_i describing the orientation of the plane.

a). Show that the drift velocity of an arbitrary stresslet is characterized by only two constants, and

b). prove that if $n_i = \delta_{i3}$ then the drift normal to the plane is proportional to only a single element of the S_{ij} tensor.

You may find the following list of third order tensors to be useful:

$$\delta_{ij}n_k \, , \; n_i n_j n_k \, , \; \delta_{ik}n_j \, , \; \varepsilon_{ijk} \, , \; \delta_{jk}n_i \, , \; \varepsilon_{jkl}n_l n_i$$

2). (20 points) An infinite cylinder is filled with fluid. Initially the fluid is at rest. At time $t=0$ the cylinder is rotated about its axis with an angular velocity which is linearly increasing in time, e.g., $u_\theta|_{r=R} = A\,t$. In this problem the velocity is only in the theta direction ($u_r = u_z = 0$) and all derivatives with respect to theta and z are zero.

a). Render the equations governing the velocity distribution dimensionless, determining appropriate scalings for the variables in the problem. How does the characteristic shear stress at the wall scale with the parameters in the problem?

b). Determine the asymptotic velocity and the wall shear stress at large times.
c). Derive the eigenvalue problem (differential equation and boundary conditions) for the decaying solution via separation of variables, but don't try to solve it. This eigenvalue problem is the same you would solve to determine how long it takes to spin up coffee in a coffee cup.

The Navier-Stokes equation for momentum in the theta direction is given by:

$$\rho\left(\frac{\partial u_\theta}{\partial t} + u_r\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r\,u_\theta}{r} + u_z\frac{\partial u_\theta}{\partial z}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta}$$
$$+\,\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(r\,u_\theta)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{\partial^2 u_\theta}{\partial z^2}\right]$$

3). (20 points) An infinite plane bounding a quiescent fluid with viscosity μ and density ρ is accelerated from rest with velocity at^2. Set up the equation governing the time-dependent velocity profile in the fluid and, using a similarity transform in canonical form, determine how the shear stress at the plane varies with time to within some unknown constant.