CHEG 544 Transport Phenomena I
Second Hour Exam

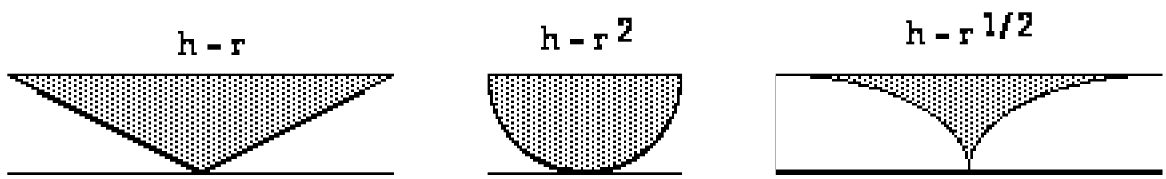
Closed Books and Notes

Problem 1). (10 points) Index Notation / Creeping Flow. Prove that the **far-field** disturbance velocity induced by any force-free and torque free particle (e.g., characterized by the stresslet $\mathbf{S_{jk}}$ which is a **physical**, **symmetric**, and **traceless** second order tensor) is purely radial. The single undetermined constant you should have the problem reduce to **could** be determined by examining the far-field velocity of some particular stresslet which you know - such as the disturbance velocity of a sphere immersed in a pure straining motion (don't do this, however!). You may find the following decaying spherical harmonics useful:

$$\frac{1}{r}\ ;\ \frac{x_i}{r^3}\ ;\ \left(\frac{x_i x_j}{r^5} - \frac{\delta_{ij}}{3 r^3}\right)\ ;\ \left(\frac{x_i x_j x_k}{r^7} - \frac{x_i \delta_{jk} + x_j \delta_{ik} + x_k \delta_{ij}}{5 r^5}\right)\ ;\ \ldots$$

Hint: Which of the relevant harmonics will contribute to the velocity and pressure for a stresslet in the **far-field limit**?

Problem 2). (20 points) Lubrication: In class and for homework we have examined the detatchment of a variety of shapes from the surface of a plane. The nature of the lubrication singularity (e.g., the force when the separation distance at the center goes to zero) depends on the geometric dependence of the gap width on radius. For the case of a sphere, h varies as r^2, and the force is infinite (e.g., zero mobility at contact). For a cone h varies as r and the force is finite. In this problem we consider the general shape, where $h^* = (r/R)^{\alpha}$ in which α is a positive constant. For what values of α will the mobility at contact be non-zero? Note: the flat disk limit with zero separation at $r = 0$ corresponds to $\alpha = \infty$ rather than $\alpha = 0$.



You may find the r momentum equation useful to get the pressure gradient:

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r}$$
$$+ \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\, v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right] + \rho\, g_r$$

Remember that in lubrication flow (at least for this problem) only a couple of terms survive!

Note: If you solve the more complicated problem of finite separations at r = 0 (don't try this here!), it turns out that a particle can come into contact with a plane even if the mobility at contact is identically zero. This happens because for some values of α the lubrication singularity is integrable in time, even if infinite (zero mobility) at contact. **For an extra point**, can you tell me what the range of α is for which this curious behavior occurs? (Hint: the other bound is a problem we've already looked at...)

Problem 3). (20 points) Consider the two-dimensional problem depicted below. We are examining the flow pattern in produced by a belt in the vicinity of a roller submerged in a viscous fluid. The belt moves with a velocity U, so the radial velocity boundary condition at $\theta = -\alpha$ is -U and that at $\theta = +\alpha$ is +U. **Determine** the velocity profile using a streamfunction formulation. **Are there any conditions** under which Moffatt Eddies can occur in this geometry for these boundary conditions? **Why or why not?**

The velocities are given by:

$$u_\theta = -\frac{\partial \psi}{\partial r} \qquad ; \qquad u_r = -\frac{1}{r}\frac{\partial \psi}{\partial \theta}$$

Recall that the general expression for a separable streamfunction in the cylindrical geometry is given by:

$$\psi_\lambda = r^{\lambda} f_\lambda(\theta)$$

where, in general, we have:

$$f_\lambda(\theta) = A_\lambda \sin(\lambda\theta) + B_\lambda \cos(\lambda\theta) + C_\lambda \sin((\lambda-2)\theta) + D_\lambda \cos((\lambda-2)\theta)$$

We also have the repeated root special cases:

$$f_0(\theta) = A_0 + B_0\,\theta + C_0 \sin(2\theta) + D_0 \cos(2\theta)$$

$$f_1(\theta) = A_1 \sin(\theta) + B_1 \cos(\theta) + C_1\,\theta \sin(\theta) + D_1\,\theta \cos(\theta)$$

$$f_2(\theta) = A_2 + B_2\,\theta + C_2 \sin(2\theta) + D_2 \cos(2\theta)$$

Hint: Think about how the radial velocity and streamfunction have to depend on r, and determine the symmetry conditions in θ!