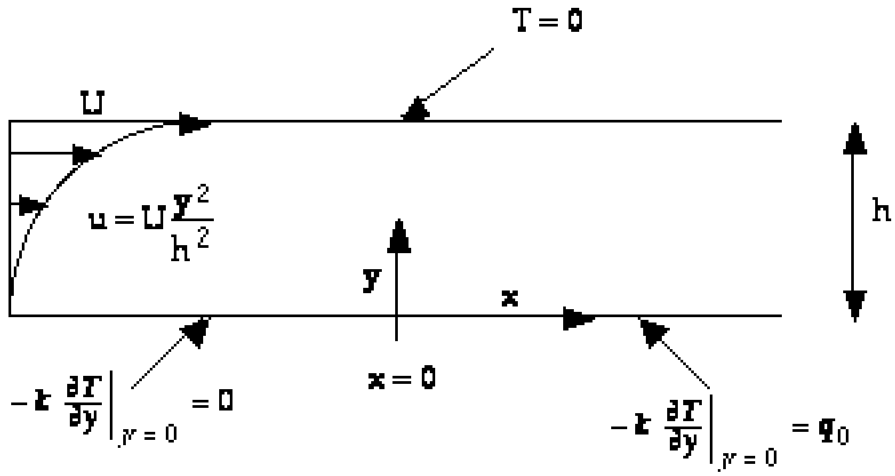
CHEG 544 Transport Phenomena I
Final Exam

Closed Books and Notes

Problem 1). (20 points) Most of this semester we have examined the asymptotic solution to complex problems. In this problem you will examine convective heat transfer in a combined plane Couette and Poiseuille flow. Suppose we have the channel depicted below. The upper wall moves with a velocity U and the lower wall is fixed. In addition to the shear flow there is a pressure driven backflow resulting in the purely quadratic dependence of velocity on y (e.g., the shear rate at the lower wall is zero):



For all x < 0 the lower wall is insulated, and for x > 0 heated at a constant rate q_0. The upper wall is maintained at a temperature T = 0. The width of the channel is h, and the thermal diffusivity is α. If we assume constant properties everywhere, the problem is governed by the differential equation:

$$u_j \frac{\partial T}{\partial x_j} = \alpha \frac{\partial^2 T}{\partial x_j^2}$$

I want you to solve the following problems, in each case carefully determining the domain of validity of the solutions:

a. The pressure gradient required to produce the desired flow.

b. The asymptotic temperature distribution for large x.

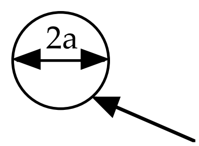
c. The eigenvalue decaying solution. Set up the Sturm-Liouville eigenvalue problem completely, and show how to calculate the coefficients, but don't actually solve the DE for the eigenfunctions.

Problem 2). (15 points) For the geometry and flow field depicted in problem 1, obtain the self-similar boundary layer solution near the entrance to the heated section. Again, set up the problem completely in canonical form obtaining the ODE and boundary conditions, but don't solve the resulting ODE. In particular determine how the temperature at the wall and boundary layer thickness depend on x. What is the domain of validity of this boundary layer solution?

Problem 3). (15 points) Matched Asymptotic Expansions. Consider a spherical thermal dipole ($T\big|_{r=a} = \lambda n_j p_j$ where n_j is the local unit normal x_j/r and p_j is the director describing the dipole orientation) immersed in a uniform flow of magnitude U as depicted below. We wish to determine the influence of convection on the temperature distribution in the limit Re << Pe << 1 (e.g., small Pe, but zero Re so that the velocity distribution is known). Your mission is to determine the solution technique!

$T\big|_{r=\infty} = 0$

$u_i\big|_{r=\infty} = U_i$



$T\big|_{r=a} = \lambda n_j p_j$

a. Starting with the convective diffusion equation given in problem 1, render the equation dimensionless and using regular perturbation expansions, set up a sequence of differential equations and boundary conditions at each order. Do this to as high an order as you think is appropriate.

b. Determine at what order we must resort to a singular perturbation expansion? What is the new length scale in the outer region?

Hint: the pure conduction solution is just the harmonic $T^{(0)} = \lambda a^2 \frac{x_i p_i}{r^3}$. Don't make this problem too difficult!

Problem 4). (10 points) Short Answer. Please answer the following questions <u>briefly</u>!

a. How does the electrophoretic mobility of DNA change if you decrease the pH (e.g., increase H^+ ion concentration) from a neutral pH?

b. Will the electroosmotic velocity increase or decrease with increasing salt concentration, and why?

c. What is the Reynolds stress and where does it come from?

d. Colloidal particles can often be knocked out of solution by "salting out" (e.g., increasing salt concentration). What's going on here?

e. We can define a turbulent Prandtl number $\nu^{(t)}/\alpha^{(t)}$ (e.g., the ratio of momentum and thermal diffusivities due to turbulence). What can you say *a priori* about this quantity?

Problem 5). (10 points) Index Notation. Consider a body of revolution *without* fore-and-aft symmetry that is fixed at the origin in a pure straining flow $u_i^\infty = E_{ij} x_j$ under creeping flow conditions.

a. What is the relationship between the force on the particle **F**, the orientation vector **p**, and the rate of strain tensor **E**? Hint: you should wind up with two terms after you have simplified it as much as possible.

b. What further reduction is possible if the body has fore-and-aft symmetry?