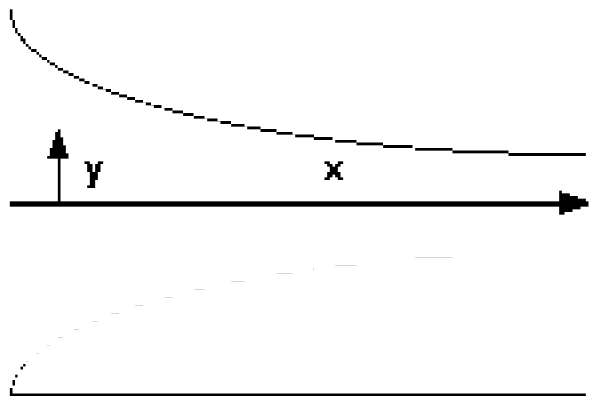
CHEG 544 Transport Phenomena I
Final Exam

Closed Books and Notes

Problem 1). (20 points) Scaling/Boundary Layers: A flat plate is inserted into a converging wind tunnel as is depicted below. The convergence of the throat of the wind tunnel leads to an external Euler flow solution of $u_e = \lambda\, x^{1/2}$ where λ is some constant.



a. What is the pressure distribution along the plate?

b. Using scaling analysis for some characteristic length L in the x-direction, determine the conditions under which the boundary layer approximation should hold. How does the boundary layer thickness vary with L (and hence x)? How does the local stress on the plate scale with L (and x)?

c. In the boundary layer limit, show that the problem for the streamfunction admits a self-similar solution. Obtain the similarity rule and variable in canonical form, as well as the boundary conditions, but don't solve for the ODE as it gets a little messy. What is the domain of validity of this solution?

Problem 2). (10 points) Lubrication: In the undergraduate transport class we demonstrate the detachment of a disk of radius R from a plane under application of a constant force F. Here we solve this problem.

a. Using scaling analysis and lubrication theory, determine how the velocity $V = dh/dt$ depends on F, μ, R, and h to within some constant (e.g. set the problem up in the lubrication limit, and render it dimensionless). How does the characteristic time depend on separation h?

b. Solve the lubrication problem to determine this constant.

c. Solve the problem for h(t) to determine the dimensionless detatchment time (e.g., the time where $1/h = 0$) for some initial separation distance h_0.

Hint: this problem is virtually identical to the squeeze flow between a sphere and a plane, except here the geometry is *much* simpler - the gap h is not a function of r! You may find the r momentum equation useful to get the pressure gradient:

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r}$$
$$+ \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\,v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right] + \rho\, g_r$$

Remember that in lubrication flow (at least for this problem) only a couple of terms survive!

Problem 3). (10 points) Index Notation.

a. Sketch out the general approach to solving for the velocity distribution for uniform flow past a sphere under creeping flow conditions using harmonics.

b. In the Midwest Mechanics lecture a couple of years ago Keith Moffatt gave a fascinating lecture on magneto-electro-hydrodynamic migration of particles. He showed that if particles were sufficiently small that inertial forces were negligible, then the migration velocity of non-conducting particles was *Bilinear* in the current density **J** (a physical vector) and the magnetic field **B** (a pseudo vector). Bilinearity means that it is linear in the product of these two vectors. Using this observation, prove that the migration velocity of a non-conducting sphere is always orthogonal to both the magnetic field and the current, and that the rotational velocity of such a sphere is identically zero. Don't make this one hard!

Problem 4). (10 points) Short Answer. Please answer the following questions briefly!

a. If a three-dimensional object is freely suspended (no net force or torque) in creeping flow, how does the disturbance velocity decay with r?

b. If the same object has a net force applied to it, how does the disturbance pressure decay with r?

c. How is the Reynolds stress defined and what does it represent?

d. Under what conditions can a flow be irrotational?

e. Show why the problem of uniform flow past a sphere at small but finite Re does not admit a regular perturbation solution to O(Re) - what goes wrong?