CBE 60544 Transport Phenomena I
First Hour Exam

Closed Books and Notes

1). (10 points) Consider a particle fixed at the origin at zero Reynolds number in the pure straining flow $u_j^\infty = E_{jk}\, x_k$ where E_{jk} is a symmetric, second order, physical tensor. We wish to examine the force and torque on the particle F_i and M_i due to E_{jk}.

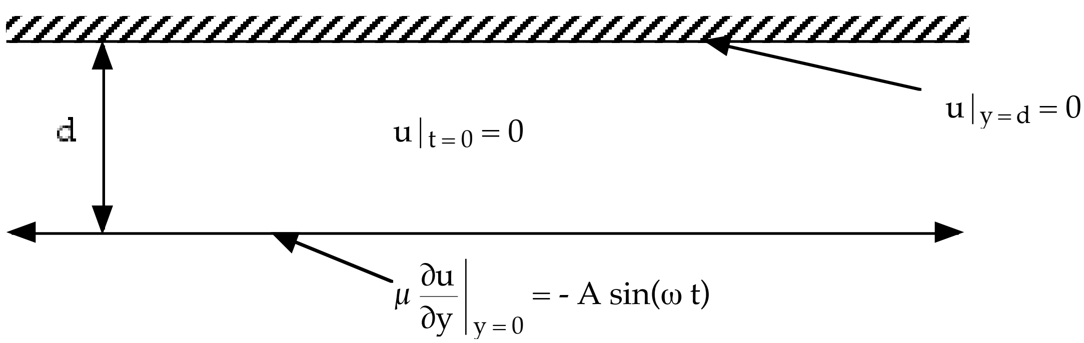
a). What is the most general tensorial relationship for an arbitrarily shaped particle between M_i and E_{jk}? What can you say in general about this tensor?

b). What is the most general tensorial relationship for an arbitrarily shaped particle between F_i and E_{jk}? What can you say in general about this tensor?

c). Show that if the particle is a sphere, the force and torque are both zero.

d). (the tricky part) Show that if the particle is a body of revolution with fore-and-aft symmetry whose orientation is specified by the director p_i, then the tensor for part (a) may be reduced to a single term (e.g., glop multiplied by one unknown scalar constant), and the tensor for part (b) is zero.

2). (20 points) A commonly used rheological measurement tool is the controlled stress rheometer, in which the applied stress is controlled and the resulting motion of a plate is used to calculate the rheological properties of the fluid. A simplified version of such a system is depicted below:



In a common operating mode the stress on the lower plate is oscillatory, which is used to measure the viscoelastic properties of polymeric fluids. The entire system is initially at rest, and at time t = 0 an oscillatory shear stress given by $\tau = -\,A \sin(\omega\, t)$ is applied to the lower wall. The velocity U of the lower wall is measured as a function of time. The in-phase component is related to the viscosity of the material (termed the loss modulus for a polymer), and the out-of-phase component to the storage modulus (e.g., the rubber band-like character of the material). Unfortunately, as you know, even for a Newtonian fluid with no viscoelasticity, inertia will give an out of phase component to the velocity if the frequency is high enough. It is this which I want you to examine.

a). Using a perturbation expansion scheme such as was done in class, determine the ratio of the magnitude of the out-of-phase to in-phase components of the velocity of the lower plate.

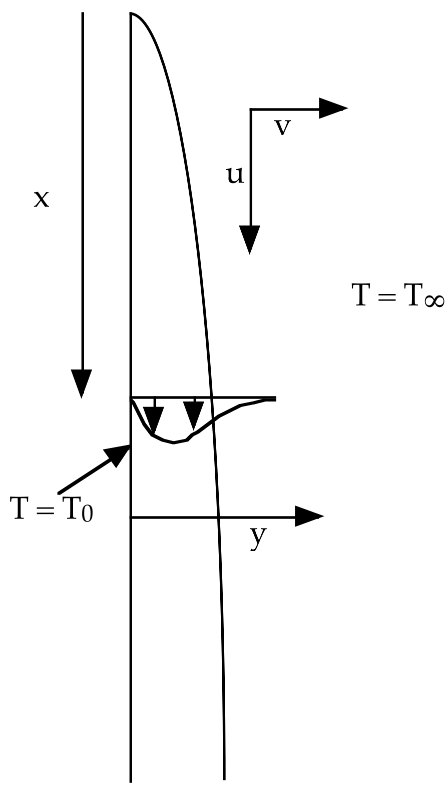
b). How long do we have to wait for initial transients to die away? What is the lead eigenvalue for the decaying problem?

3). (20 points) Scaling analysis and self-similarity. For homework you scaled the equations governing natural convection from a point source of energy. In this problem we determine the characteristic velocity produced by the draft off of a cold window pane as depicted below. We take the temperature of the pane (e.g., y = 0) to be T_0, and that at infinity to be T_∞. The velocities vanish at the pane, and u = 0 far from the pane.

a. Choosing L as the characteristic length in the x-direction, determine the characteristic thickness of the convection plume, and the magnitude of the velocity. Remember: don't solve the equations, just scale them!

b. Using the results from part (a), or doing simple affine stretching again, show that the problem admits a self-similar solution and give the transformed variables in canonical form. You don't need to get the transformed differential equations.

The flow is governed by:



$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \beta g\left(T - T_\infty\right)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u|_{y=0} = v|_{y=0} = u|_{y=\infty} = 0;$$

$$T|_{y=0} = T_0 \ ; \ T|_{y=\infty} = T_\infty$$