CHEG 544 Transport Phenomena I
Second Hour Exam

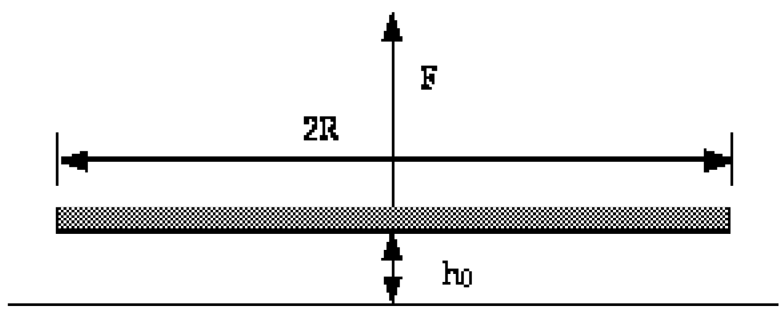
Closed Books and Notes

Problem 1). (10 points) Creeping Flow/Index Notation: Consider a sphere of radius a freely suspended (e.g., no net force or torque on the sphere, it is free to rotate and translate with the fluid) in a general shear flow $u_i^\infty = \Gamma_{ij}\, x_j$ at zero Re.

a. Using the concept of linearity and physical arguments, show that the disturbance velocity produced by the sphere is the same as that of a sphere fixed at the origin in the pure straining flow $u_i^\infty = 0.5\,(\Gamma_{ij} + \Gamma_{ji})\, x_j$.

b. Using index notation, sketch out how you would determine this disturbance velocity.

Problem 2). (20 points) Lubrication: In the undergrad fluids course we demonstrate how the initial separation distance controls the detachment of a disk of radius R from a plane, as depicted below.



a. If the upward force on the disk is given by F (ignore all buoyancy and gravitational effects!), use lubrication theory and scaling analysis to determine how long a disk with initial separation $h_0 << R$ takes to detach from the plane to within some unknown O(1) numerical factor. Detachment occurs when the separation distance becomes much greater than the initial separation, e.g., $R >> h(t) >> h_0$.

b. Solve the lubrication problem (rather than just scaling!) to figure out the exact detachment time.

You may find the r momentum equation useful to get the pressure gradient:

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r}$$
$$+\,\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\, v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right] + \rho\, g_r$$

Remember that in lubrication flow (at least for this problem) only a couple of terms survive!

Problem 3). (20 points) Consider the two-dimensional problem depicted below. Two walls separated by an angle 2α (note that α is not small - this is not the lubrication problem you did for homework!) are opening outward, so that $u_\theta \big|_{\theta = \pm\alpha} = \pm\, r\, d\alpha/dt$. We wish to determine the velocity profile using a streamfunction formulation. The velocities are given by:

$$u_\theta = -\frac{\partial\psi}{\partial r} \qquad ; \qquad u_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta}$$

Recall that the general expression for a separable streamfunction in the cylindrical geometry is given by:

$$\psi_\lambda = r^{\lambda} f_\lambda(\theta)$$

where, in general, we have:

$$f_\lambda(\theta) = A_\lambda \sin(\lambda\theta) + B_\lambda \cos(\lambda\theta) + C_\lambda \sin((\lambda\text{-}2)\theta) + D_\lambda \cos((\lambda\text{-}2)\theta)$$

We also have the repeated root special cases:

$$f_0(\theta) = A_0 + B_0\, \theta + C_0 \sin(2\theta) + D_0 \cos(2\theta)$$

$$f_1(\theta) = A_1 \sin(\theta) + B_1 \cos(2\theta) + C_1\, \theta \sin(\theta) + D_1\, \theta \cos(\theta)$$

$$f_2(\theta) = A_2 + B_2\, \theta + C_2 \sin(2\theta) + D_2 \cos(2\theta)$$

Using the above relations, solve for the streamfunction and velocity distribution. Hint: Think about how the theta velocity and streamfunction have to depend on r, and about symmetry relations for u_θ and Ψ in θ.