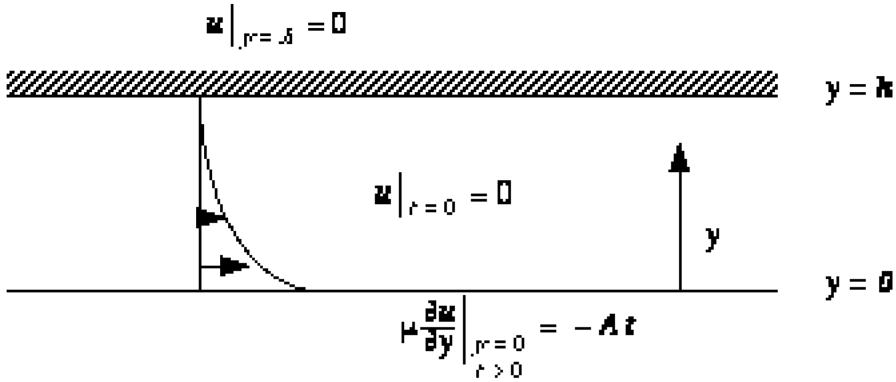
CHEG 544 Transport Phenomena I
Final Exam

Closed Books and Notes

Problem 1). (20 points) Unidirectional Flows/Scaling Analysis: The Couette viscometer depicted below is designed such that the lower wall exerts a stress on the fluid whose magnitude is linearly increasing in time.



a. Write down the differential equation and boundary conditions governing this problem, and render them dimensionless.

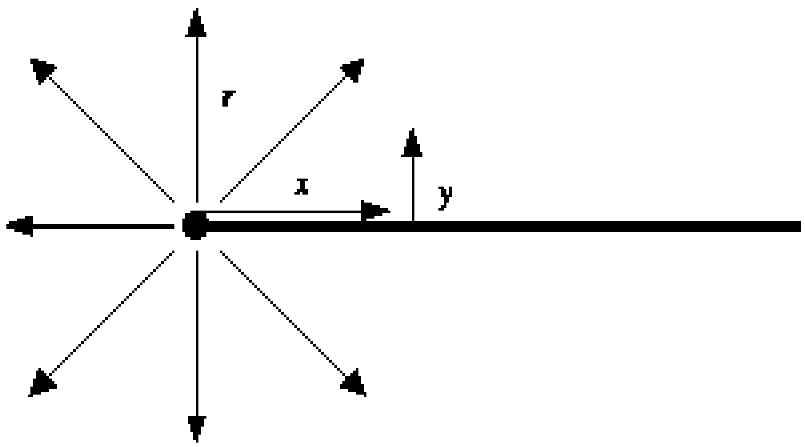
b. Determine the asymptotic velocity distribution at large times (e.g., everything except the part that exponentially decays in time). Explicitly determine the velocity of the lower plate.

c. Solve for the decaying solution. You may leave the coefficients in terms of integrals.

d. At short times this problem admits a boundary-layer solution. Determine the similarity rule and variable, and transformed ODE and boundary conditions, but don't solve the ODE. What is the velocity of the lower plate (to within an unknown constant)?

e. Putting together the solutions to parts b-d, sketch out the dimensionless velocity of the lower plate as a function of dimensionless time, marking the domain of validity of each approximate solution. When including the results of part c, only keep the leading term in the expansion.

Problem 2). (20 points) Scaling/Boundary Layers: Two-dimensional radial source flow is produced when fluid is squirted out radially from a cylindrical surface as depicted below. The undisturbed velocity distribution is simply the flow rate per unit extension into the paper divided by a circumference, e.g., $u_r = (Q/W)/(2\pi r)$. A flat plate is inserted into this flow as is depicted below.



a. If the external flow (e.g., the undisturbed radial source flow) is at high Re, what is the pressure gradient along the plate? Is it an adverse or favorable pressure gradient?

b. Using scaling analysis for some characteristic length L in the x-direction along the plate, determine the conditions under which the boundary layer approximation should hold. How does the boundary layer thickness vary with L (and hence x)? How does the local stress on the plate scale with L (and x)?

c. In the boundary layer limit, show that the problem for the streamfunction admits a self-similar solution. Obtain the similarity rule and variable in canonical form, as well as the boundary conditions and ODE. Discuss the domain of validity of this solution.

Problem 3). (10 points) Short Answer. Please answer the following questions <u>briefly</u>!

a. How is the Reynolds stress defined and what does it represent?

b. If the thermal energy per unit volume is given by $\rho C_p T$, how would you define the heat transfer analogue of the Reynolds stress, and what does it mean?

c. Prove that turbulence must increase drag in flow through a pipe.

d. A body of revolution with orientation specified by a director p_i is settling in a fluid at zero Re under the action of a force F_i. What is the most general relationship for its velocity U_i as a function of orientation?

e. Does the answer to part d change at finite Re? Briefly justify your answer.