CHEG 60544 Transport Phenomena I
First Hour Exam

Closed Books and Notes

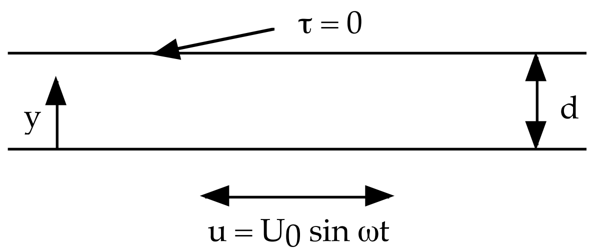
1). (15 points) A sphere is freely suspended (e.g., no forces or torques are exerted on it) in an infinite fluid undergoing the linear shear flow $u_i = A_{ij} x_j$ at zero Reynolds number.

a. Determine the most general relationship for the sphere's angular velocity Ω_i as a function of the rate of strain tensor A_{ij}.

b. Using this, prove that the angular velocity of a sphere suspended in any pure straining flow (e.g., a symmetric rate of strain tensor $A_{ij} = A_{ji}$) is zero.

c. What is the angular velocity of a sphere in the simple shear flow $u_i = G\, \delta_{i1}\, x_2$? Be specific!

2). (45 points) Consider the system depicted below. A film of depth d, density ρ and viscosity μ is on top of a plate which is oscillated back and forth in the x direction with amplitude $u = U_0 \sin \omega t$. The fluid above the film is air, so the boundary condition at y = d is just the zero shear stress condition. You may take the flow to be unidirectional. We are interested in the asymptotic behavior at large times (e.g., after *initial* transients have died away).



a). Render the governing equation and boundary conditions dimensionless. What is the dimensionless group that appears in the problem?

b). Solve for the velocity distribution for all values of this dimensionless parameter, leaving the problem in complex form.

c). Asymptotic limit 1: low frequencies. Explicitly solve for the velocity distribution in the limit of low dimensionless oscillation frequencies. What is the amplitude of the shear stress at the lower wall, and what is the amplitude of the velocity at the upper surface in this limit? (Hint: this is most easily solved via a regular perturbation expansion rather than taking the limit of the solution to part b. Carry it to first order in the perturbation parameter).

d). Asymptotic limit 2: high frequencies. Solve for an approximate velocity distribution in the limit of high frequencies. **Estimate** the amplitude of the shear stress at the lower wall and the magnitude of the oscillatory velocity at the upper free surface.

e). How long do we have to wait for initial transients to die away? What is the lead eigenvalue for the decaying problem?