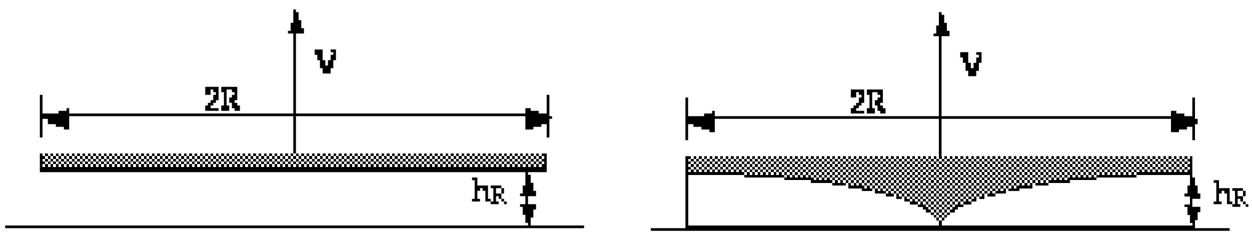
**CHEG 60544 Transport Phenomena I**
**Second Hour Exam**

**Closed Books and Notes**

Problem 1). (20 points) Lubrication: You are asked to determine the effect of the shape of the contact region of a disk on its mobililty: the force required for detatchment from a plane. Consider the two geometries depicted below. In case (a) on the left, the gap width is at a uniform value $h_R$ over the entire disk. In case (b) on the right, the gap has the same value $h_R$ at the outer edge, but goes to zero at the center as $h = h_R\,(r/R)^{1/2}$. Ignoring the effects of inertia and buoyancy (gravity) in the lubrication limit, by what factor does the contact singularity at the center increase the force necessary to move the plates with a constant initial velocity?



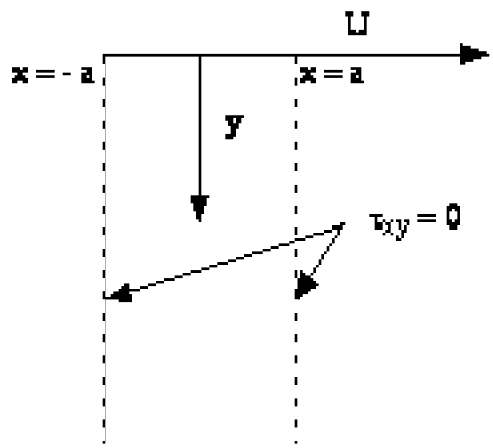

You may find the r momentum equation useful:

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r}$$
$$+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\,v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right] + \rho\, g_r$$

Remember that in lubrication flow (at least for this problem) only a couple of terms survive!

Problem 2). (20 points) Creeping Flow/Index Notation: Consider a sphere of radius a undergoing solid body rotation with angular velocity $\Omega_i$ in an unbounded fluid at rest. Using index notation, determine the disturbance velocity produced by the sphere. Remember that the spherical harmonics are simply the derivatives of the fundamental harmonic 1/r.

3). (20 points) Consider creeping flow in a slot of infinite length as depicted below. The flow is driven by a lid moving with velocity U, and the walls of the slot ($x = \pm a$) may be considered to be stress free. There is no flow through the boundaries of the slot, so the walls are a streamline. Solve for the velocity profile in terms of the streamfunction.




Recall that the general solutions to the biharmonic equation that are separable in cartesian coordinates are of the form: $e^{imx} e^{my}$, $xe^{imx} e^{my}$, and $ye^{imx} e^{my}$ where m is, in general, complex.

Hint: the velocity and the streamfunction should die away as $y \to \infty$, and you might want to use eigenfunction expansions in the x-direction. This should suggest what functional representation you should take for the x and y dependence of the streamfuction (e.g., trigonometric, hyperbolic, or exponential).