CHEG 60544 Transport Phenomena I
Final Exam

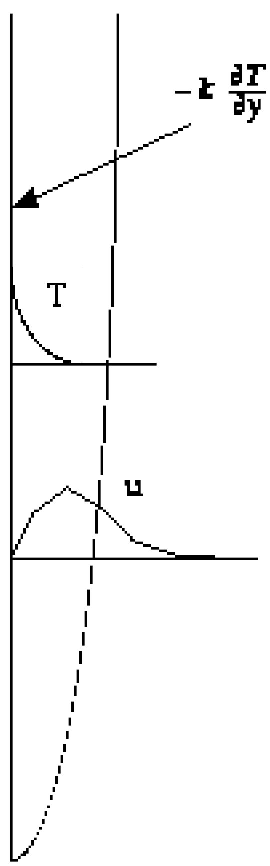
Closed Books and Notes

Problem 1). (20 points) Scaling/Boundary Layers. Consider natural convection from a heated plane as depicted below. A fluid is heated by a constant heat flux q at y = 0. Because the fluid expands, it rises upward (positive x-direction) due to natural convection. For small heat fluxes, the change in density with temperature is significant only in the buoyancy term of the equations of motion. The governing differential equations and boundary conditions are given below:

a. Scale these equations to determine the characteristic magnitude of the temperature of the plate ΔT_c, the vertical velocity U_c and the boundary layer thickness δ for a plate of height L as a function of the heat flux q and the physical properties of the fluid.

b. Under what conditions will the problem admit a boundary layer solution? Give the answer in terms of q, L, and physical properties.

c. Show that the problem admits a self-similar solution in the boundary layer limit. How do u, T, and the boundary layer thickness depend on x? What is the domain of validity in x of this self-similar solution? (Note: you don't have to get the transformed ODE's or BC's, but you can if you want to as a check of your work.)



$$-k \left.\frac{\partial T}{\partial y}\right|_{y=0} = q$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \beta g (T - T_0)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$

$$u|_{y\to\infty} \to 0;\quad P|_{y\to\infty} \to 0;\quad T|_{y\to\infty} \to T_0$$

$$-k\left.\frac{\partial T}{\partial y}\right|_{y=0} = q;\quad u|_{y=0} \to 0;\quad v|_{y=0} \to 0$$

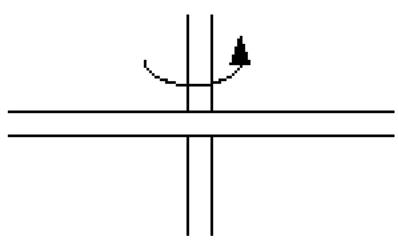
Problem 2). (20 points) Lubrication. In a recent paper (Davies & Stokes, J. Rheol 2005) a fundamental problem with current parallel-plate viscometers was identified. As you know, the viscosity of a fluid is measured in a parallel-plate viscometer by rotating one plate at a known angular velocity and measuring the resultant torque on the other plate. The viscosity calculation, however, requires accurate knowledge of the gap width between the plates. In a modern instrument it is possible to set the gap accurately automatically -provided- you have the reference point of where the two plates are in contact: e.g., where the gap is zero. For an instrument such as is in Prof. Hill's lab (Rheometrics Ares), you just push a button, the upper plate decends at a constant velocity of $50\mu m/s$, and the gap is considered to be zero when the measured upward thrust on the upper plate exceeds 10^4 dynes, providing the reference point for all subsequent gaps. This would be fine if there were no fluid in the gap (e.g., a vacuum), but as you know the viscosity of air is non-zero (1.8×10^{-4} poise). In this problem we analyze what the effect of air is on the gap zeroing of 3cm radius parallel-plates.

a. For a normal force detection threshold F, approach velocity V, air viscosity μ, and plate radius R, calculate the error in the gap set zero.

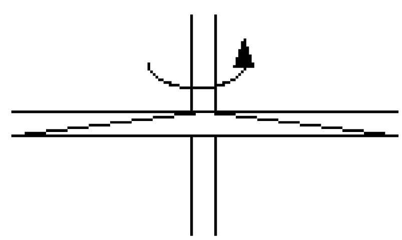
b. For the numbers given above, calculate the numerical value of the error in microns. Is it practical to reduce this error by an order of magnitude by decreasing V? Be specific!

c. If we set the gap to $200\mu m$ using this incorrect zero (e.g., actual gap differs from $200\mu m$ due to the zero error), what is the corresponding error in the viscosity measurement μ_{exp}/μ of a fluid?

d. A co-worker suggests that this may not be a problem if we use a small angle cone-and-plate geometry rather than the parallel-plate geometry. Using dimensional analysis alone (e.g., don't get the O(1) number that takes all the work), give a quantitative estimate if this is likely to be true for a 1 degree (0.0175 radians) cone.



Parallel-Plate Geometry



Cone-and-Plate Geometry

Problem 3). (20 points) Index Notation/Creeping Flow. A general quadratic shear flow at zero Re is given below. Pressure driven flow through a tube or a channel are examples of such a flow. This is also the local multivariable Taylor series expansion of a completely general flow field. The parameters U_i, Γ_{ij}, and A_{ijk} are constants, and are physical vectors and tensors.

$$u_i^\infty = U_i + \Gamma_{ij} x_j + \tfrac{1}{2} A_{ijk} x_j x_k$$

a. What constraint does continuity place on Γ_{ij} and A_{ijk}? Are there any required symmetries in A_{ijk}?

b. What is the undisturbed pressure distribution p^∞ produced by this flow field?

c. Consider a sphere fixed at the origin in this flow field. What is the most general expression for the resulting torque? Reduce this as much as possible.

d. What is the most general expression for the force on the sphere? Again, reduce this as much as possible (Hint: you can get it down to two terms!).

e. What is the general form of the disturbance pressure produced by the sphere (e.g., the first step in calculating the complete pressure and velocity distributions)? You should be able to get this down to four unknown λ's. Where would you go from here in calculating the disturbance velocity distribution?