CHEG 60544 Transport Phenomena I
First Hour Exam

Closed Books and Notes

1). (15 points) A body of revolution whose orientation is specified by the director p_i is fixed at the origin in an infinite fluid undergoing the general linear shear flow $u_i = A_{ij} x_j$ at zero Reynolds number.

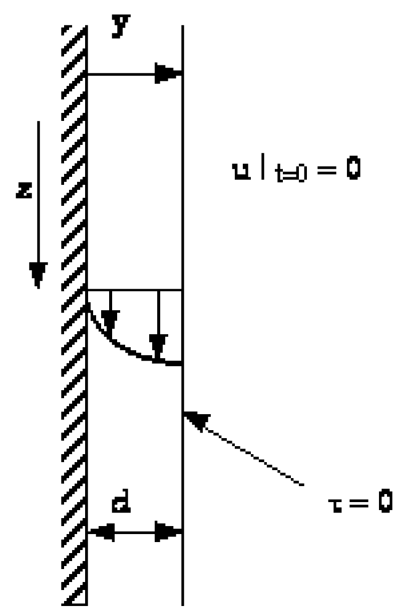
a. Determine the simplest (fewest constants!) general relationship for the force F_i and torque M_i on the particle as a function of the rate of strain tensor A_{ij}.

b. What simplifications are possible if the body posesses fore-and-aft symmetry?

The following third order tensors may be of use:

$$\varepsilon_{ijk},\ \varepsilon_{ijl}p_lp_k,\ \varepsilon_{ilk}p_lp_j,\ \varepsilon_{ljk}p_lp_i,\ p_i\delta_{jk},\ p_j\delta_{ik},\ p_k\delta_{ij},\ p_ip_jp_k$$

2). (30 points) Consider the geometry depicted below. A film of thickness d, density ρ and viscosity μ is coating a vertical plane. Initially, the film is at rest ($u|_{t=0} = 0$). For $t > 0$ gravity acts on the film, causing it to accelerate, eventually reaching some asymptotic steady velocity distribution. The fluid outside the film is air, so the boundary condition at y = d is just the zero shear stress condition. You may take the flow to be unidirectional.



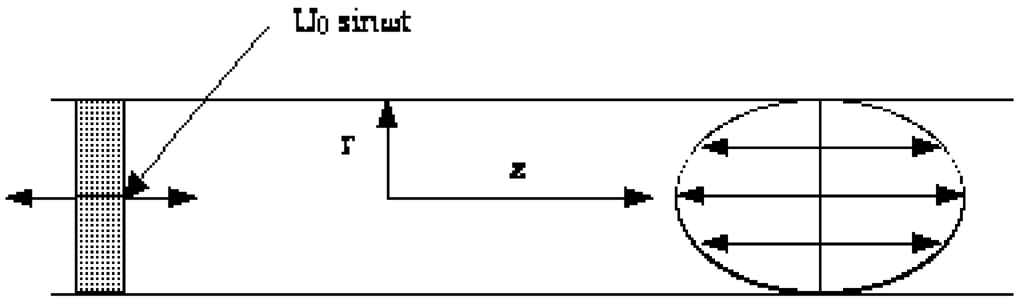
a). Render the governing equation and boundary conditions dimensionless. What is the characteristic time scale, and what is its physical interpretation?

b). Solve for the velocity distribution at steady-state.

c). Develop an expression for the velocity distribution valid at all times using the separation of variables technique, obtaining both eigenfunctions and eigenvalues. Leave the expression for the coefficients in integral form.

d). Short time asymptote: Show that the problem admits a self-similar solution for very short times, obtaining the similarity rule and variable in canonical form. How does the shear stress at the wall vary with time?

3). (15 points) In our laboratory we often use an oscillatory syringe pump to drive oscillatory flow in a tube. Here we look at the pressure gradient resulting from such a flow. Consider the geometry depicted below. An oscillatory piston (plunger) in a tube of radius R moves with velocity $U_0 \sin\omega t$. The fluid has a kinematic viscosity ν. Far from the piston, the flow may be regarded as unidirectional in the z direction, although obviously stiill a function of r and t, and the motion of the piston and fluid will produce some pressure gradient $-\partial p/\partial z = G(t)$.



a). Write down the governing equation and boundary conditions, and render them dimensionless. What dimensionless parameter appears in the problem, and what is its physical significance?

b). In the limit of low frequencies, solve for the velocity distribution and pressure gradient G(t).

c). Using a regular perturbation expansion, obtain the first inertial correction to G(t).

You may find the following equation helpful:

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial z^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho\, g_z$$