CHEG 544 Transport Phenomena I
Second Hour Exam

Closed Books and Notes

Problem 1). (20 points) The Jeffrey-Hamel Problem. Consider the two-dimensional problem depicted below. Fluid drains from a trough with interior angle 2α at a rate Q/W where W is the extension of the trough in the third dimension. We wish to determine the velocity profile using a streamfunction formulation. The velocities are given by:

$$u_\theta = -\frac{\partial\psi}{\partial r} \qquad ; \qquad u_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta}$$

Recall that the general expression for a separable streamfunction in the cylindrical geometry is given by:

$$\psi_\lambda = r^{\lambda} f_\lambda(\theta)$$

where, in general, we have:

$$f_\lambda(\theta) = A_\lambda \sin(\lambda\theta) + B_\lambda \cos(\lambda\theta) + C_\lambda \sin((\lambda-2)\theta) + D_\lambda \cos((\lambda-2)\theta)$$

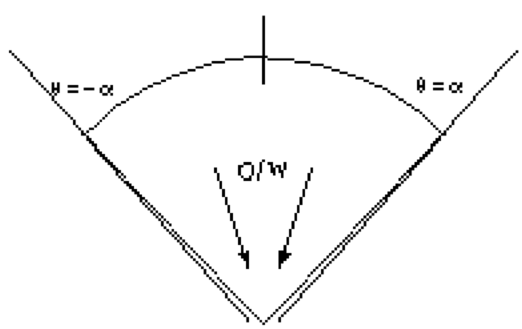
We also have the repeated root special cases:

$$f_0(\theta) = A_0 + B_0\,\theta + C_0 \sin(2\theta) + D_0 \cos(2\theta)$$

$$f_1(\theta) = A_1 \sin(\theta) + B_1 \cos(2\theta) + C_1\,\theta \sin(\theta) + D_1\,\theta \cos(\theta)$$

$$f_2(\theta) = A_2 + B_2\,\theta + C_2 \sin(2\theta) + D_2 \cos(2\theta)$$

Using the above relations, solve for the streamfunction and velocity distribution. Hint: Think about how the radial velocity and streamfunction have to depend on r, and determine the symmetry conditions in θ!



Problem 2). (20 points) Index Notation / Creeping Flow. The fundamental singularity of low Reynolds number flows is the Stokeslet, the far-field disturbance velocity induced by any point force vector F_j. Here we examine this profile.

a. Derive the **far-field** pressure and velocity distributions resulting from the Stokeslet F_j to within an unknown constant.

b. Explicitly show how this constant could be determined from a force balance - but don't actually do it as this would be messy!

You may find the following decaying spherical harmonics useful:

$$\frac{1}{r}\ ;\ \frac{x_i}{r^3}\ ;\ \left(\frac{x_i x_j}{r^5}-\frac{\delta_{ij}}{3r^3}\right)\ ;\ \left(\frac{x_i x_j x_k}{r^7}-\frac{x_i\,\delta_{jk}+x_j\,\delta_{ik}+x_k\,\delta_{ij}}{5r^5}\right)\ ;\ \ldots$$

Hint: Which of the relevant harmonics will contribute to the velocity and pressure for a Stokeslet in the **far-field limit**? Also, don't forget continuity!

Problem3). (20 points) Lubrication. Consider the geometry below. Two sheets are drawn with a velocity U between two rollers of radius R whose surfaces are separated by a distance 2 b_0. The idea is to fill the space between the sheets with a viscous fluid of viscosity μ, and all the action takes place in a lubrication layer between the rollers, at least in the limit $b_0/R << 1$. Here we analyze this problem.

a. By scaling the flow equations in the gap, show how the force/width on the rollers F/L scales with the separation distance between the rollers and the other parameters of the problem.

b. The sheet sandwich detatches from the rollers downstream at a point x_d and final gap width 2 b_f > 2 b_0 when the pressure again returns to zero. Develop an implicit integral relation for the ratio b_f/b_0, and show that it doesn't depend on any of the other parameters of the problem.

c. Develop an integral relationship for dimensionless pressure and force, but don't try to evaluate it (it would have to be done numerically, although that's pretty easy with a computer).

Hint: the gap geometry in the lubrication limit is given by $b = b_0 + \frac{1}{2}\frac{x^2}{R}$ where x is the distance along the gap from the point of minimum separation.