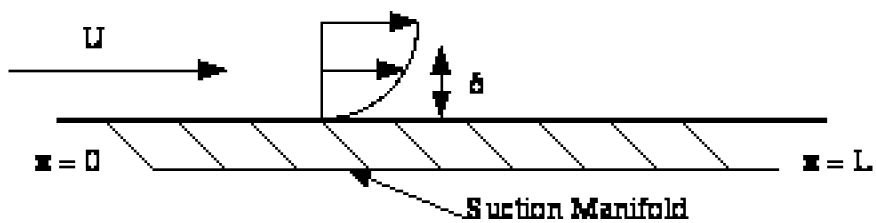
CBE 60544 Transport Phenomena I
Final Exam

Closed Books and Notes

Problem 1). (10 points) Scaling/Boundary Layers: At high Reynolds number, boundary layer control is critical! One way of controlling boundary layer growth is via suction - essentially sucking out the boundary layer (region of slow flow) to delay boundary layer separation. You are assigned the job of designing an experiment to study the effect of boundary layer suction on boundary layer growth. You have a plate of length L and width W in a wind tunnel with velocity U, as depicted below. What is the characteristic magnitude of the total suction flow rateQ to result in an O(1) change in the boundary layer thickness (e.g., a change comparable to the thickness in the absence of suction)?



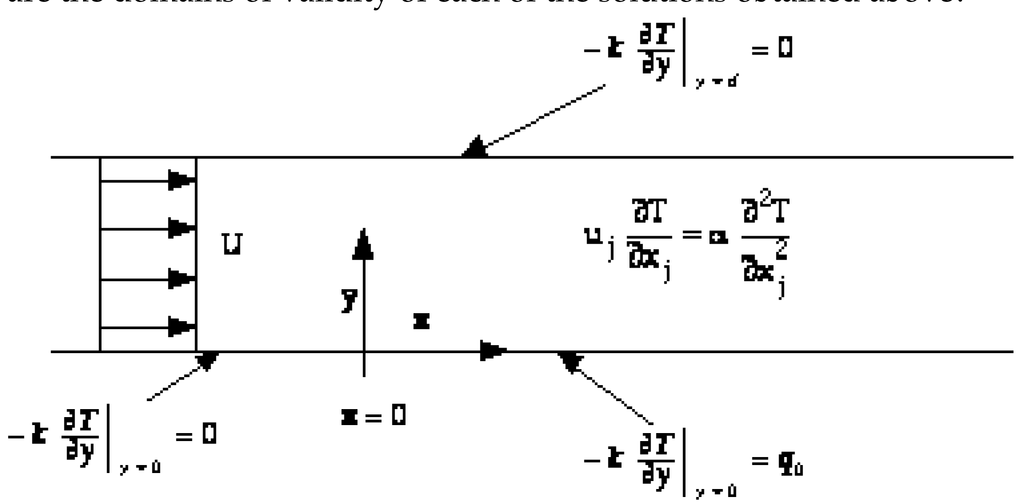
Problem 2). (20 points) In a fabrication process a sheet of material of thickness d is convected into a heating section with velocity U as depicted below. If, for all $x > 0$, the heater delivers a constant flux q_0 to the bottom side of the sheet (e.g., $y = 0$), solve the following:

a. The asymptotic temperature distribution at large x.

b. The decaying eigenvalue solution.

c. The self-similar boundary layer solution close to the entrance of the heated section.

d. What are the domains of validity of each of the solutions obtained above?



Problem 3). (10 points) Lubrication/Scaling: Consider the detachment of a disk of radius R from a plane under application of a constant force F. Using scaling analysis and lubrication theory, determine how the velocity $V = dh/dt$ depends on F, μ, R, and h to within some constant (e.g. set the problem up in the lubrication limit, and render it dimensionless). How does the characteristic detachment time depend on the initial separation h_0? Note that I'm not asking you to -solve- the problem, just determine the characteristic time scale!

Hint: You may find the r momentum equation useful to get the pressure gradient:

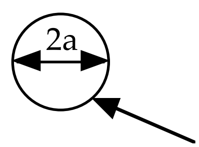
$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r}$$
$$+\,\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\,v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right] + \rho\, g_r$$

Remember that in lubrication flow (at least for this problem) only a couple of terms survive!

Problem 4). (10 points) Matched Asymptotic Expansions. Consider a spherical thermal dipole ($T\big|_{r=a} = \lambda n_j p_j$ where n_j is the local unit normal x_j/r and p_j is the director describing the dipole orientation) immersed in a uniform flow of magnitude U as depicted below. We wish to determine the influence of convection on the temperature distribution in the limit Re << Pe << 1 (e.g., small Pe, but zero Re so that the velocity distribution is known). Your mission is to determine the solution technique!

$$T\big|_{r=\infty} = 0$$

$$u_i\big|_{r=\infty} = U_i$$



$$T\big|_{r=a} = \lambda n_j p_j$$

a. Starting with the convective diffusion equation given in problem 2, render the equation dimensionless and, using regular perturbation expansions, set up a sequence of differential equations and boundary conditions at each order. Do this to as high an order as you think is appropriate.

b. Determine at what order we must resort to a singular perturbation expansion? What is the new length scale in the outer region?

Hint: the pure conduction solution is just the harmonic $T^{(0)} = \lambda a^2 \frac{x_i p_i}{r^3}$. Don't make this problem too difficult!

Problem 5). (10 points) Short Answer. Please answer the following questions briefly!

a. If a three-dimensional object is freely suspended (no net force or torque) in creeping flow, how does the disturbance velocity decay with r?

b. If the same object has a net force applied to it, how does the disturbance pressure decay with r?

c. How is the Reynolds stress defined and what does it represent?

d. Under what conditions can a flow be irrotational?

e. We can define a turbulent Prandtl number $\nu^{(t)}/\alpha^{(t)}$ (e.g., the ratio of momentum and thermal diffusivities due to turbulence). What can you say *a priori* about this quantity?